1 Asymptotic Bound Practice

Prove that for any $\epsilon > 0$ we have $\log x \in O(x^\epsilon)$.

2 Bounding Sums

Let $f(\cdot)$ be a function. Consider the equality

$$\sum_{i=1}^{n} f(i) \in \Theta(f(n)),$$

Give a function $f_1$ such that the equality holds, and a function $f_2$ such that the equality does not hold.

3 In Between Functions

Prove or disprove: If $f : \mathbb{N} \rightarrow \mathbb{N}$ is any positive-valued function, then either (1) there exists a constant $c > 0$ so that $f(n) \in O(n^c)$, or (2) there exists a constant $\alpha > 1$ so that $f(n) \in \Omega(\alpha^n)$. 
4 Recurrence Relation Practice

Derive an asymptotic tight bound for the following $T(n)$. Cite any theorem you use.

(a) $T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \sqrt{n}$.

(b) $T(n) = T(n-1) + c^n$ for constants $c > 0$.

(c) $T(n) = 2T(\sqrt{n}) + 3$, and $T(2) = 3$. 