CS 170 DIS 01

Released on 2018-09-03

1 Squaring vs multiplying: matrices

The square of a matrix A is its product with itself, AA.

(a) Show that five multiplications are sufficient to compute the square of a 2×2 matrix.

(b) What is wrong with the following algorithm for computing the square of an $n \times n$ matrix?

”Use a divide-and-conquer approach as in Strassen’s algorithm, except that instead of getting 7 subproblems of size $n/2$, we now get 5 subproblems of size $n/2$ thanks to part (a). Using the same analysis as in Strassen’s algorithm, we can conclude that the algorithm runs in $\Theta(n^{\log_2 5})$ time.”

(c) In fact, squaring matrices is no easier than multiplying them. Show that if $n \times n$ matrices can be squared in $\Theta(n^c)$ time, then any $n \times n$ matrices can be multiplied in $\Theta(n^c)$ time.

2 Find the missing integer

An array A of length N contains all the integers from 0 to N except one (in some random order). In this problem, we cannot access an entire integer in A with a single operation. The elements of A are represented in binary, and the only operation we can use to access them is “fetch the jth bit of $A[i]$”. Using only this operation to access A, give an algorithm that determines the missing integer by looking at only $O(N)$ bits. (Note that there are $O(N \log N)$ bits total in A, so we can’t even look at all the bits). Assume the numbers are in bit representation with leading 0s.

3 Complex numbers review

(a) Write each of the following numbers in the form $\rho(\cos\theta + i \sin\theta)$ (for real ρ and θ):

(i) $-\sqrt{3} + i$

(ii) The three third roots of unity

(iii) The sum of your answers to the previous items

(b) Let sqrt(x) represent one of the complex square roots of x, so that $(\text{sqrt}(x))^2 = x$. What are the possible values of sqrt(sqrt(-1))?

You can use any notation for complex numbers, e.g., rectangular, polar, or complex exponential notation.