CS 170 Dis 9

Released on 2018-10-29

1 Maximal Matching

Let $G = (V, E)$ be a (not necessarily bipartite) undirected graph. A *maximal matching*, M, is a matching in which no edge can be added while keeping it a matching. Show that the size of any maximal matching is at least half the size of a maximum matching M^*.

2 Bipartite Vertex Cover

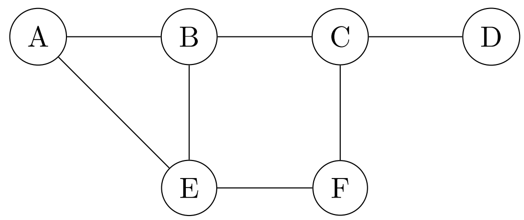
A vertex cover of an undirected graph $G = (V, E)$ is a subset of the vertices which touches every edge. In other words, a subset $S \subset V$ such that for each edge $\{u, v\} \in E$, one or both of u, v are in S.

Show that the problem of finding the minimum vertex cover in a *bipartite* graph reduces to maximum flow. Prove that your reduction is correct.

Hint: use the max-flow min-cut theorem.

3 Reducing Vertex Cover to Set Cover

In the minimum vertex cover problem, we are given an undirected graph $G = (V, E)$ and asked to find the smallest set $U \subseteq V$ that "covers" the set of edges E. In other words, we want to find the smallest set U such that for each $(u, v) \in E$, either u or v is in U (U is not necessarily unique). For example, in the following graph, $\{A, E, C, D\}$ is a vertex cover, but not a minimum vertex cover. The minimum vertex covers are $\{B, E, C\}$ and $\{A, E, C\}$.



Recall the following definition of the minimum Set Cover problem: Given a set U of elements and a collection $S_1, \ldots, S_m$ of subsets of U, what is the smallest collection of these sets whose union equals U? So, for example, given $U := \{a, b, c, d\}$, $S_1 := \{a, b, c\}$, $S_2 := \{b, c\}$, and $S_3 := \{c, d\}$, a solution to the problem is the collection of S_1 and S_3.

Give an efficient reduction from the Minimum Vertex Cover Problem to the Minimum Set Cover Problem.

4 Midterm Discussion

What did you find most challenging on the midterm? Are there any problems in particular you would like to discuss?