# CS 170 DIS 10

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## 1 NP Basics

Assume A reduces to B in polynomial time. In each part you will be given a fact about one of the problems. What will you know about the other problem from each fact? *(You can answer each part in one sentence.)*

1. A is in **P**.
2. B is in **P**.
3. A is **NP**-hard.
4. B is **NP**-hard.

## 2 Hitting Set

In the Hitting Set Problem, we are given a family of finite integer sets $\{S_1, S_2, \ldots, S_n\}$ and a budget $b$, and we wish to find an integer set $H$ of size $\leq b$ which intersects every $S_i$, if such an $H$ exists. In other words, we want $H \cap S_i \neq \emptyset$ for all $i$.

Show that the Hitting Set Problem is NP-complete.

## 3 Reliable Network

Reliable Network is the following problem: We are given two $n \times n$ matrices (a cost matrix $d_{ij}$ and a connectivity requirement matrix $r_{ij}$) and also a budget $b$. We want to find a graph $G = (\{1, ..., n\}, E)$ such that the total cost of all edges (i.e. $\sum_{(i,j)\in E} d_{ij}$) is at most $b$ and there are exactly $r_{ij}$ vertex-disjoint paths between any two distinct vertices i and j.

Show that Reliable Network is NP-Complete.

## 4 Dominating Set

A dominating set of a graph $G = (V, E)$ is a subset D of V, such that every vertex not in $D$ is a neighbor of at least one vertex in $D$.

Let the Minimum Dominating Set problem be the task of determining whether there is a dominating set of size $\leq k$.

Show that the Minimum Dominating Set problem is NP-Hard. You may assume for this question that all graphs are connected.