CS 170 DIS 10

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1 NP Basics

Assume A reduces to B in polynomial time. In each part you will be given a fact about one of the problems. What will you know about the other problem from each fact? (You can answer each part in one sentence.)

1. A is in \( \mathbf{P} \).
2. B is in \( \mathbf{P} \).
3. A is \( \mathbf{NP} \)-hard.
4. B is \( \mathbf{NP} \)-hard.

2 Hitting Set

In the Hitting Set Problem, we are given a family of finite integer sets \( \{S_1, S_2, \ldots, S_n\} \) and a budget \( b \), and we wish to find an integer set \( H \) of size \( \leq b \) which intersects every \( S_i \), if such an \( H \) exists. In other words, we want \( H \cap S_i \neq \emptyset \) for all \( i \).

Show that the Hitting Set Problem is \( \mathbf{NP} \)-complete.
3 Reliable Network

Reliable Network is the following problem: We are given two \( n \times n \) matrices (a cost matrix \( d_{ij} \) and a connectivity requirement matrix \( r_{ij} \)) and also a budget \( b \). We want to find a graph \( G = (\{1, ..., n\}, E) \) such that the total cost of all edges (i.e. \( \sum_{(i,j) \in E} d_{ij} \)) is at most \( b \) and there are exactly \( r_{ij} \) vertex-disjoint paths between any two distinct vertices \( i \) and \( j \).

Show that Reliable Network is NP-Complete.

4 Dominating Set

A dominating set of a graph \( G = (V, E) \) is a subset \( D \) of \( V \), such that every vertex not in \( D \) is a neighbor of at least one vertex in \( D \).

Let the Minimum Dominating Set problem be the task of determining whether there is a dominating set of size \( \leq k \).

Show that the Minimum Dominating Set problem is NP-Hard. You may assume for this question that all graphs are connected.