CS 170 HW 4

Due on 2018-09-23, at 9:59 pm

1 (★) Study Group

List the names and SIDs of the members in your study group.

2 (★★) Maximum Subarray Sum

Given an array of n integers, the maximum subarray is the contiguous subarray (potentially empty) with the largest sum. Design a linear algorithm to find the sum of the maximum subarray. For example the maximum subarray of $[-2, 1, -3, 4, -1, 2, 1, -5, 4]$ is $[4, -1, 2, 1]$, whose sum is 6.

Please give a three-part solution of the following format:

(a) **Clearly** describe your algorithm. You can include the pseudocode optionally.

(b) Write a proof of correctness.

(c) Write a runtime analysis.

3 (★) MST Basics

For each of the following statements, either prove or supply a counterexample. Always assume $G = (V, E)$ is undirected and connected. Do not assume the edge weights are distinct unless specifically stated.

(a) Let e be any edge of minimum weight in G. Then e must be part of some MST.

(b) If e is part of some MST of G, then it must be a lightest edge across some cut of G.

(c) If G has a cycle with a unique lightest edge e, then e must be part of every MST.

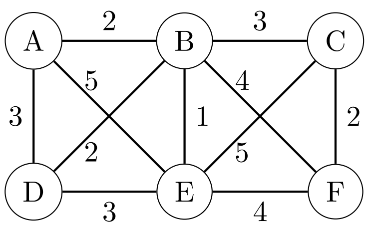
(d) For any $r > 0$, define an r-path to be a path whose edges all have weight less than r. If G contains an r-path from s to t, then every MST of G must also contain an r-path from s to t.

4 (★) Prim's Algorithm

A popular alternative to Kruskal's algorithm is Prim's algorithm, in which the intermediate set of edges X always forms a subtree, and S is chosen to be the set of this tree's vertices. We can think of Prim's algorithm as greedily processing one vertex at a time, adding it to S. The pseudocode below gives the basic outline of Prim's algorithm. See the book for a detailed example of a run of the algorithm.

```
S = {v}
X = {}
While S ≠ V:
    Choose t ∈ V\S, s ∈ S such that weight(s,t) is minimized
    X = X ∪ {(s,t)}
    S = S ∪ {t}
Return X
```

(a) Run Prim's algorithm on the following graph, starting from A, stating which node you processed and which edge you added at each step .



(b) Prim's algorithm is very similar to Dijkstra's in that a vertex is processed at each step which minimizes some cost function. These algorithms also produce similar outputs: the union of all shortest paths produced by a run of Dijkstra's algorithm forms a tree. However, the trees they produce aren't optimizing for the same thing. To see this, give an example of a graph for which different trees are produced by running Prim's algorithm and Dijkstra's algorithm. In other words, give a graph where there is a shortest path from a start vertex A using at least one edge that doesn't appear in any MST.

5 (★) Divide and Conquer for MST?

Is the following algorithm correct? If so, prove it. Otherwise, give a counterexample and explain why it doesn't work.

procedure FINDMST(G: graph on n vertices)
  If $n = 1$ return the empty set
  $T_1 \leftarrow$ FindMST(G_1: subgraph of G induced on vertices $\{1, \ldots, n/2\}$)
  $T_2 \leftarrow$ FindMST(G_2: subgraph of G induced on vertices $\{n/2+1, \ldots, n\}$)
  $e \leftarrow$ cheapest edge across the cut $\{1, \ldots, \frac{n}{2}\}$ and $\{\frac{n}{2}+1, \ldots, n\}$.
  return $T_1 \cup T_2 \cup \{e\}$.