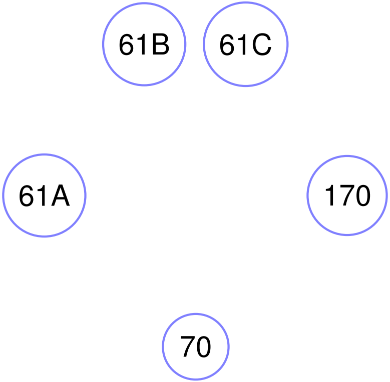
CS170 - Lecture 6

Sanjam Garg
UC Berkeley

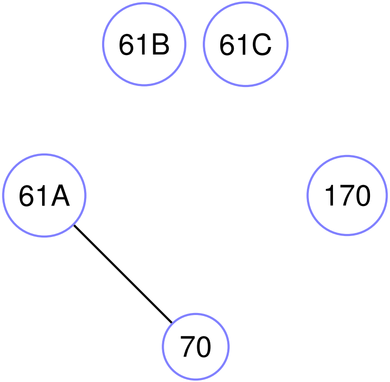
Today

1. Graphs
2. Depth First Search
3. Reachability

Scheduling: coloring.



Scheduling: coloring.



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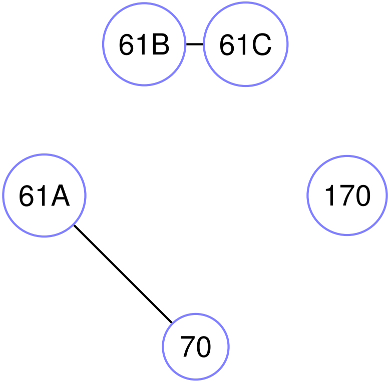

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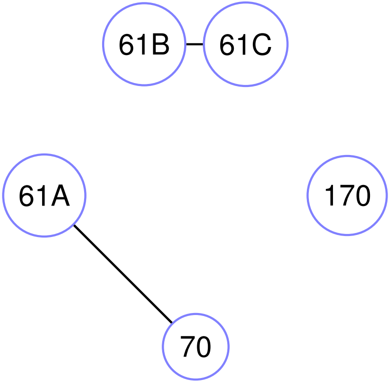

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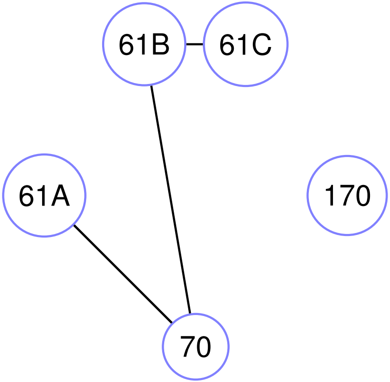

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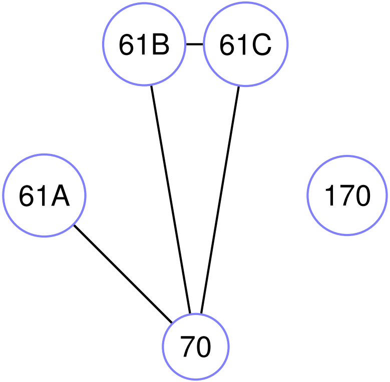

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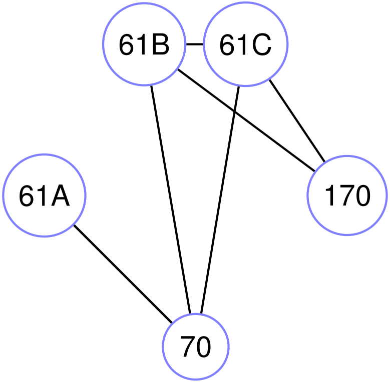

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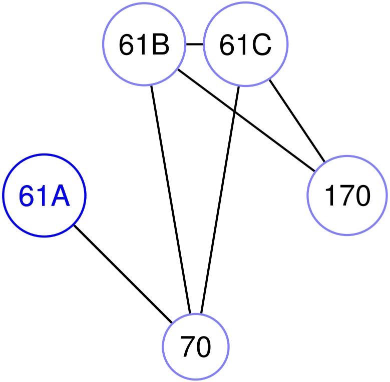

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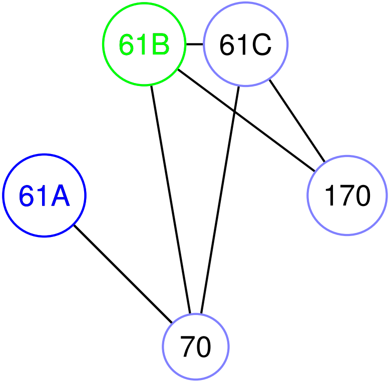

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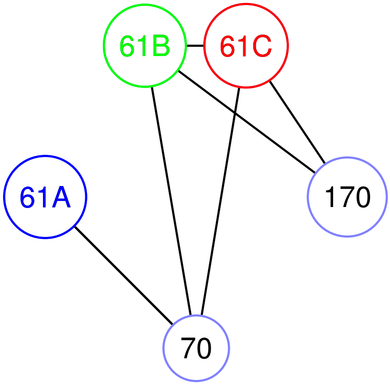

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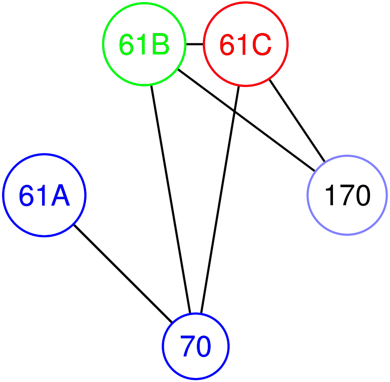

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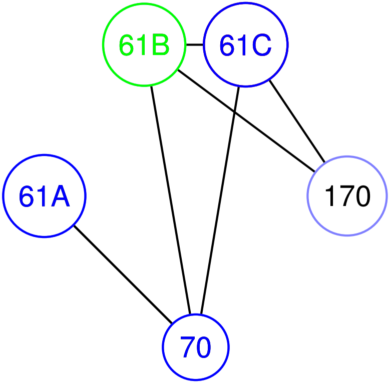

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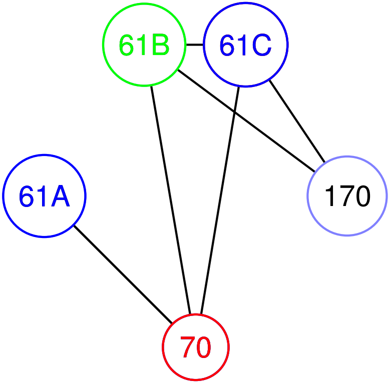

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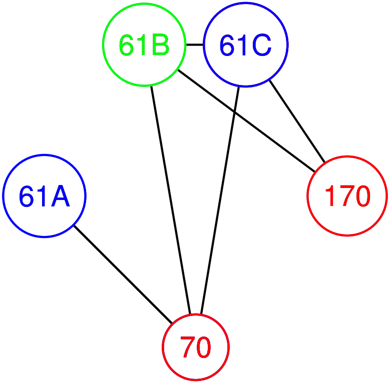

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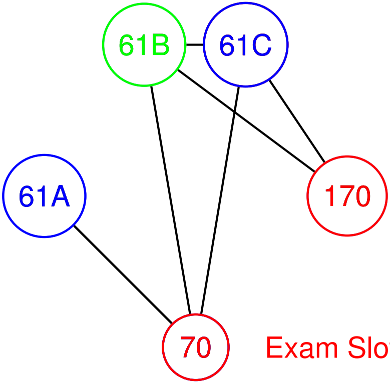


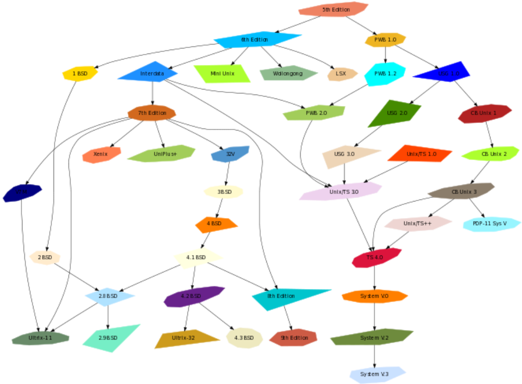
Exam Slot 1.

Exam Slot 2.

Exam Slot 3.

Directed acyclic graphs.

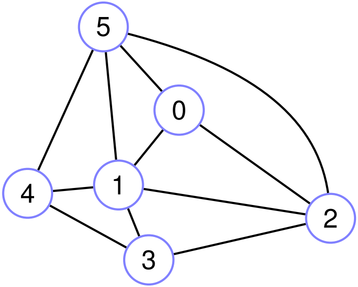
Heritage of Unix.



Object Oriented Graphs
Stephen North, 3/19/93

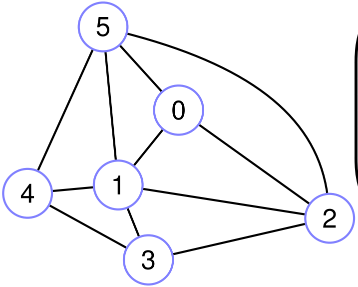
From http://www.graphviz.org/content/crazy.

Graph $G = (V, E)$.



$$V = \{0, 1, 2, 3, 4, 5\}$$
$$E = \{(0,1), (0,2), (0,5), (1,3) \ldots\}$$

Graph $G = (V, E)$.



Matrix Representation.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

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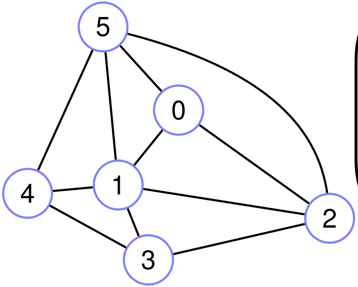

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Adjacency List

$0 :\quad 1, 2, 5$

$1 :\quad 0, 2, 3, 4, 5$

$2 :\quad 0, 1, 3, 5$

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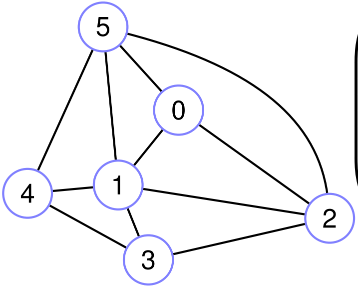
$4 :\quad 1, 3, 5$

$5 :\quad 0, 1, 2, 4$

Graph $G = (V, E)$.



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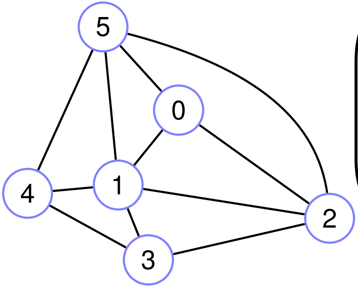
	Matrix	Adj. List
Edge (u, v)?		

Graph $G = (V, E)$.



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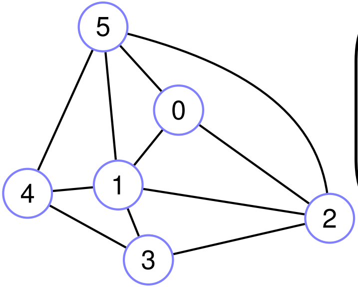
	Matrix	Adj. List
Edge (u, v)?	$O(1)$	

Graph $G = (V, E)$.



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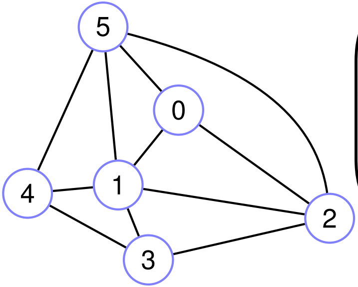
	Matrix	Adj. List
Edge (u, v)?	$O(1)$	$O(d)$
Neighbors of u		

Graph $G = (V, E)$.



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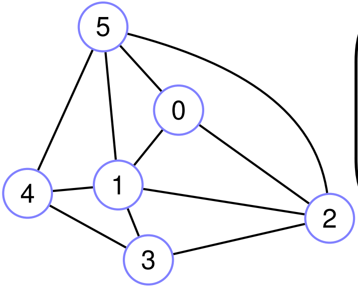
	Matrix	Adj. List
Edge (u, v)?	$O(1)$	$O(d)$
Neighbors of u	$O(\vert V \vert)$	

Graph $G = (V, E)$.



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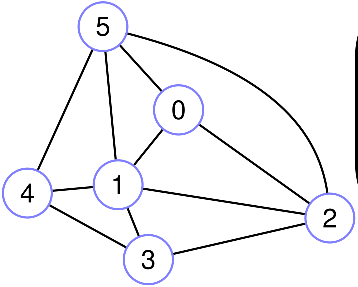
	Matrix	Adj. List
Edge (u, v)?	$O(1)$	$O(d)$
Neighbors of u	$O(\lvert V \rvert)$	$O(d)$
Space		

Graph $G = (V, E)$.



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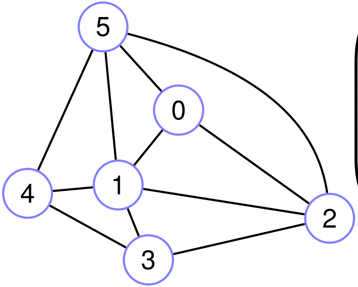
	Matrix	Adj. List
Edge (u, v)?	$O(1)$	$O(d)$
Neighbors of u	$O(\lvert V \rvert)$	$O(d)$
Space	$O(\lvert V \rvert^2)$	

Graph $G = (V, E)$.



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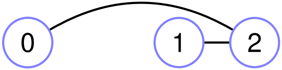
	Matrix	Adj. List
Edge (u, v)?	$O(1)$	$O(d)$
Neighbors of u	$O(\|V\|)$	$O(d)$
Space	$O(\|V\|^2)$	$O(\|E\|)$

Test your understanding..



Adjacency list of node 0?

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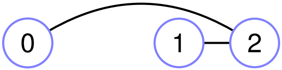


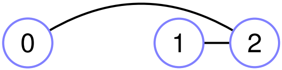
Adjacency list of node 0?

(A) $0 : 1$

(B) $0 : 1, 2$

(C) $0 : 2$

Test your understanding..



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(C)

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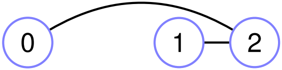


Adjacency list of node 0?

(A) $0:1$

(B) $0:1,2$

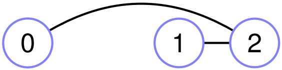
(C) $0:2$

(C)

How many edges?

(A) 2

Test your understanding..



Adjacency list of node 0?

(A) 0 : 1

(B) 0 : 1, 2

(C) 0 : 2

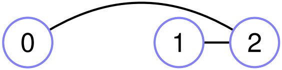
(C)

How many edges?

(A) 2

Total length of adacency lists?

Test your understanding..



Adjacency list of node 0?

(A) $0 : 1$

(B) $0 : 1, 2$

(C) $0 : 2$

(C)

How many edges?

(A) 2

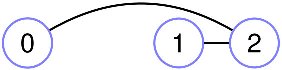
Total length of adacency lists?

(A) 2

(B) 3

(C) 4

Test your understanding..



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How many edges?

(A) 2

Total length of adacency lists?

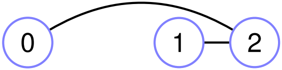
(A) 2

(B) 3

(C) 4

(C)

Test your understanding..



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(C) $0 : 2$

(C)

How many edges?

(A) 2

Total length of adacency lists?

(A) 2

(B) 3

(C) 4

(C) 2 entries for each edge!

Exploring a maze.

Theseus: Wants to find the minatour in the maze.

Exploring a maze.

Theseus: Wants to find the minatour in the maze.

Exploring a maze.

Theseus: Wants to find the minatour in the maze.

Theseus has access to a Ball of Thread and a Chalk!

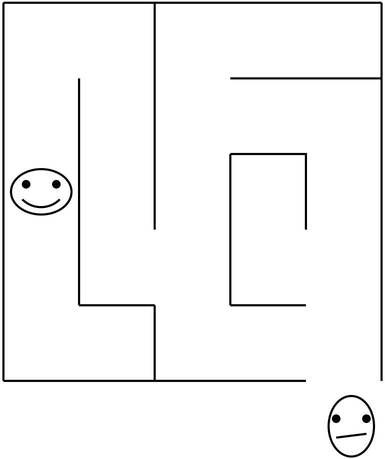
Exploring a maze.

Theseus: Wants to find the minatour in the maze.

Theseus has access to a Ball of Thread and a Chalk!

Explore a room: Mark room with chalk.
For each exit.
  Look through exit. If marked, next exit.
  Otherwise go in room unwind thread.
    Explore that room.
Wind thread to go back to “previous” room.

Where is the minatour?



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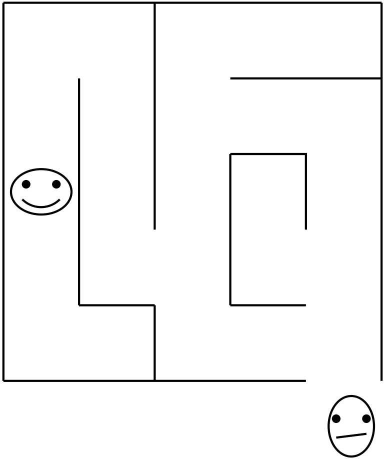

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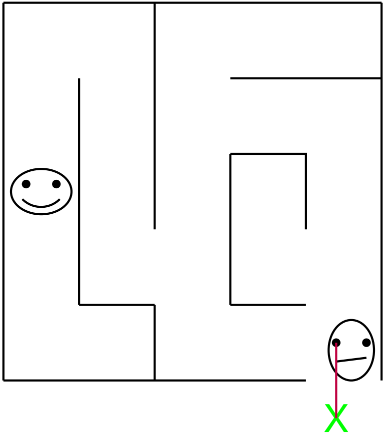

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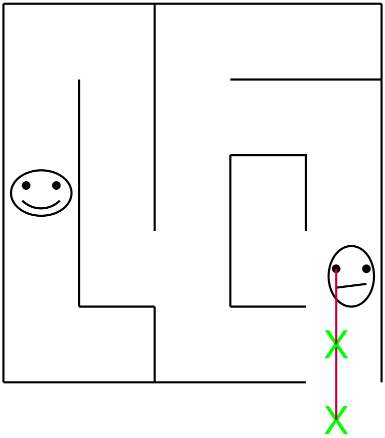

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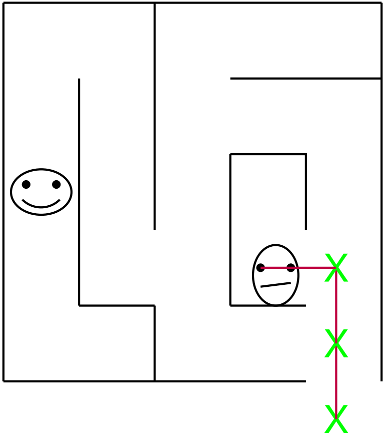

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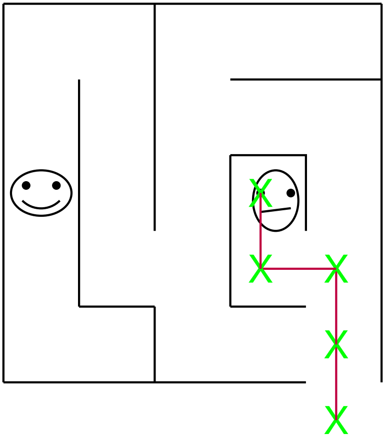

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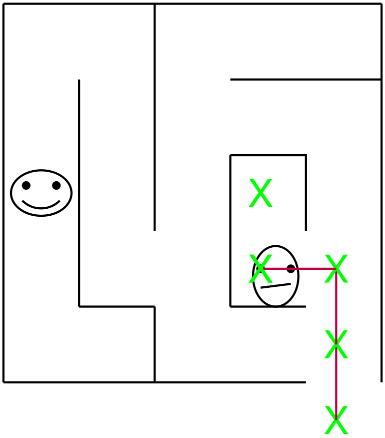

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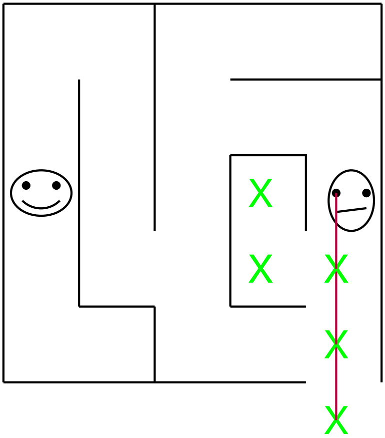

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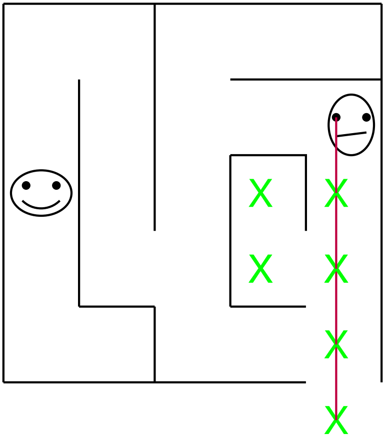

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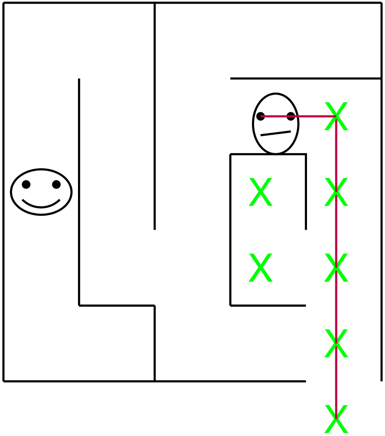

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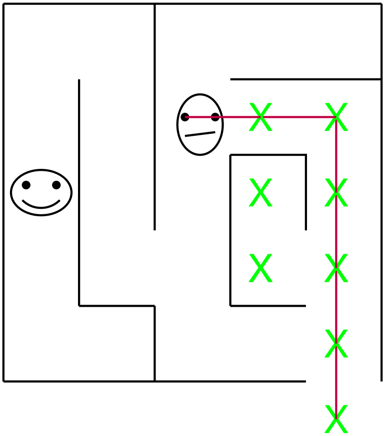
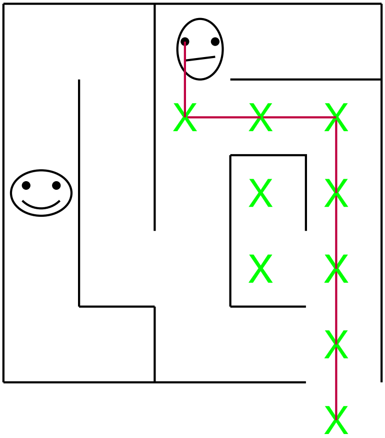


Where is the minatour?



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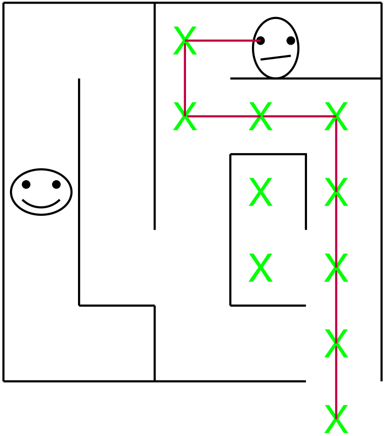

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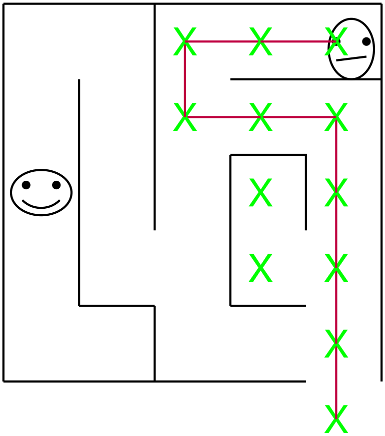

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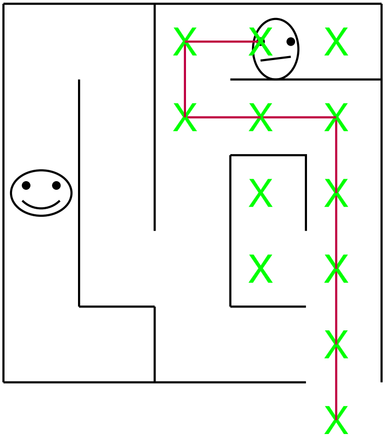

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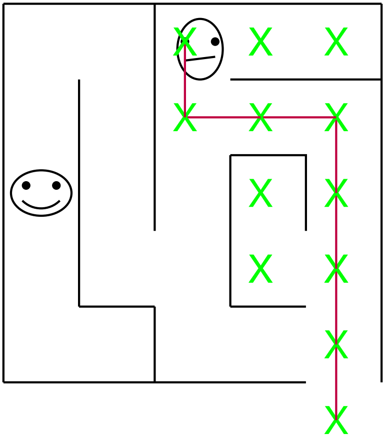

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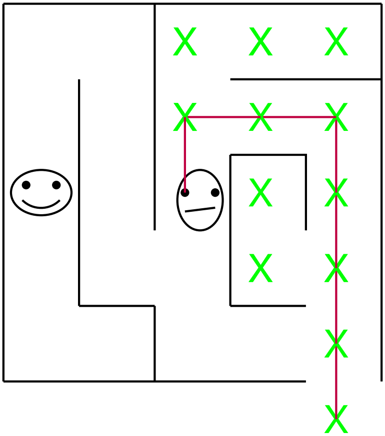

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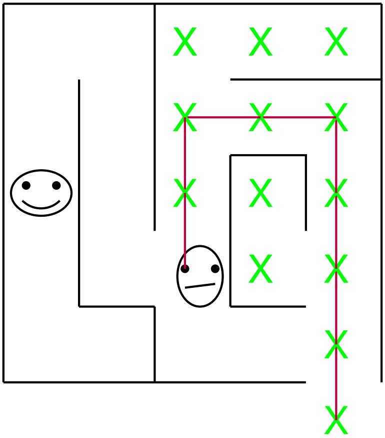

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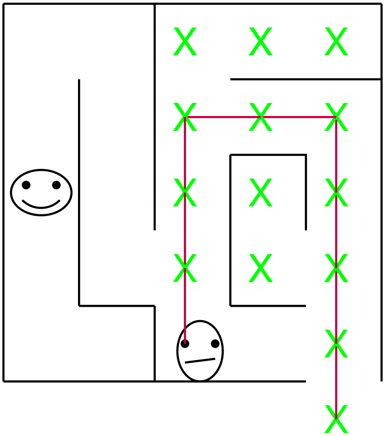

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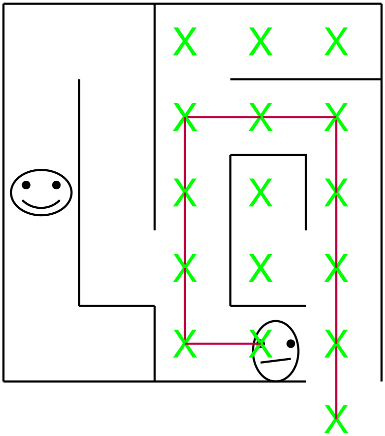

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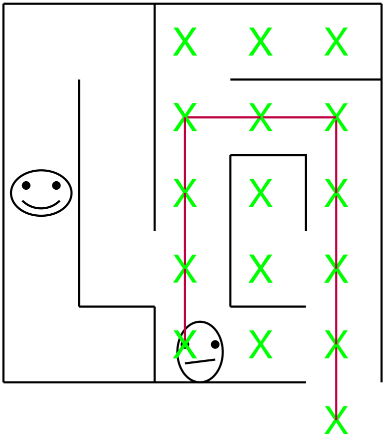

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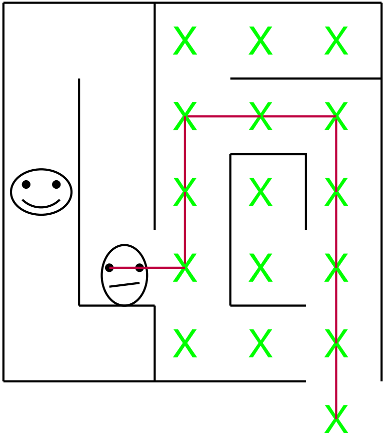

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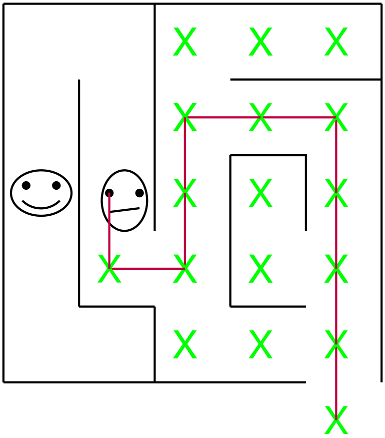

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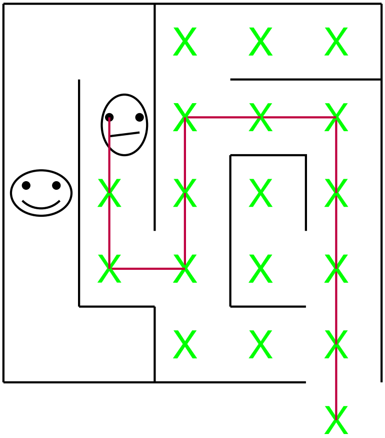

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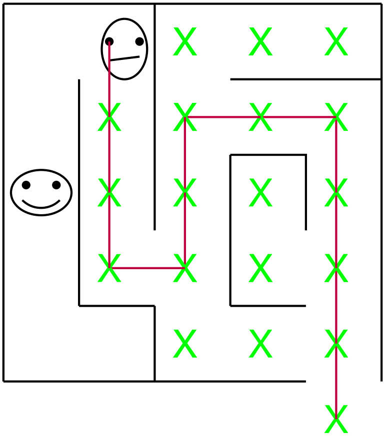

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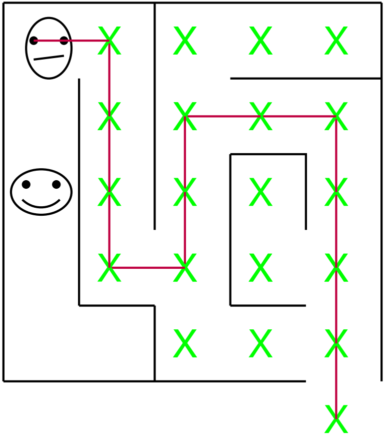

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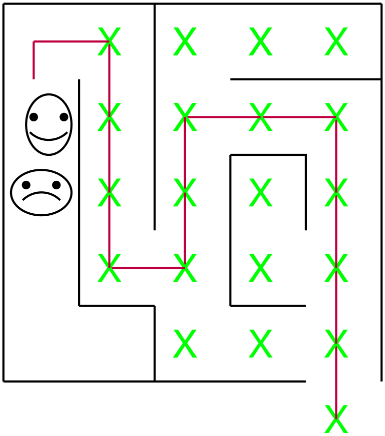

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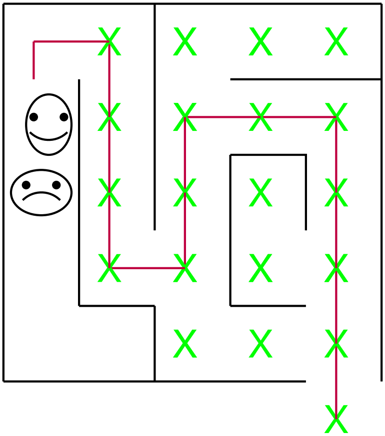

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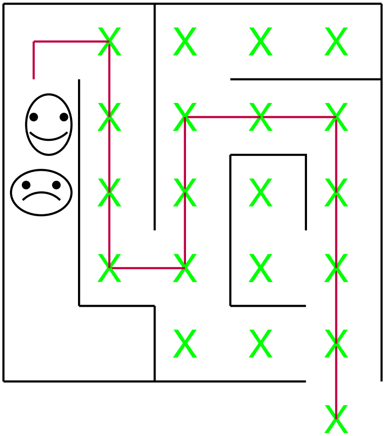

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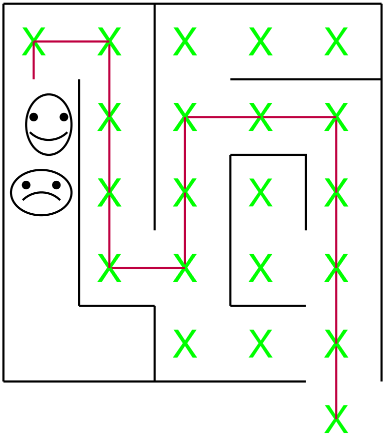
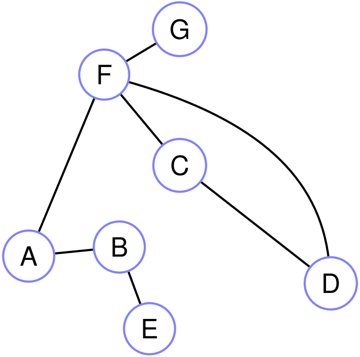


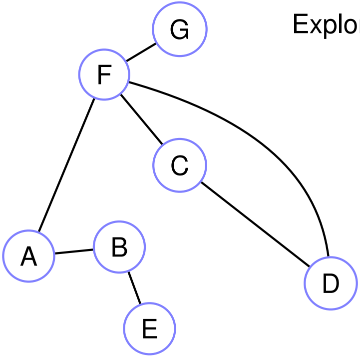
Reachability problem in a Graph.

Problem: Find out which nodes are reachable from A.
Need digital analogues of the chalk and ball of thread.
We will use array (visited) for chalk and stack for thread.

Explore.



Explore.



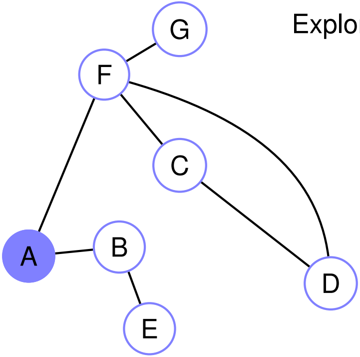
Explore(v):

1. Set visited[v] := true
2. for each edge (v,w) in E
3. if not visited[w]: Explore(w).

Chalk.

Stack is Thread.

Explore.



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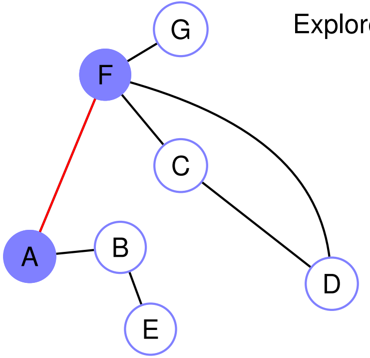

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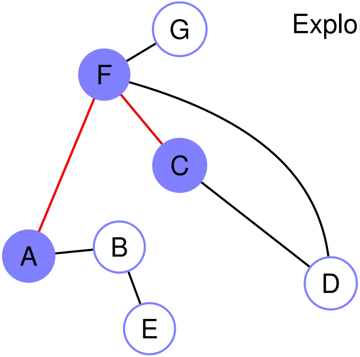

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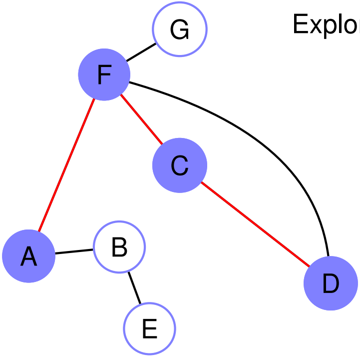
Chalk.

Stack is Thread.

Explore.



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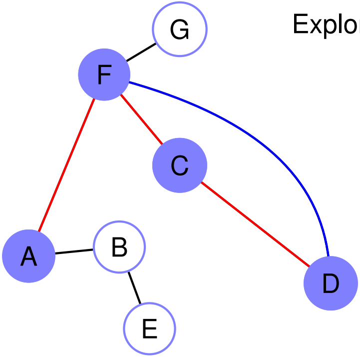

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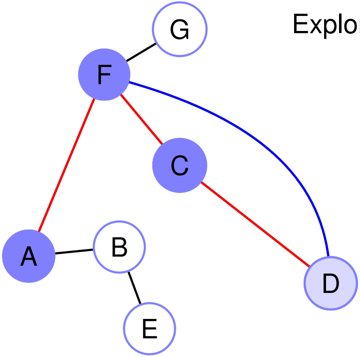

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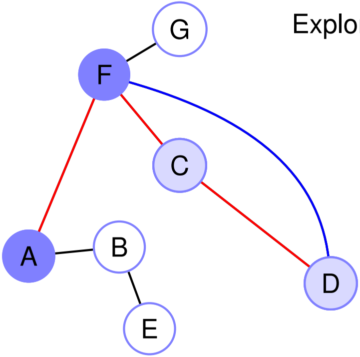

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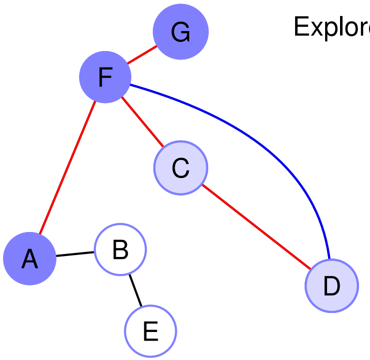

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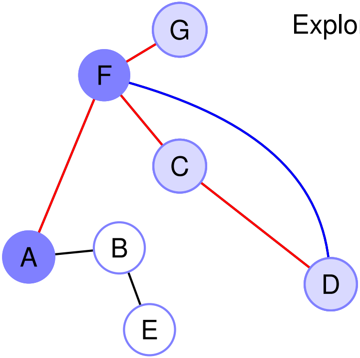

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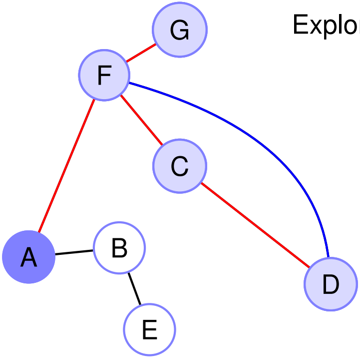
Chalk.
Stack is Thread.

Explore.



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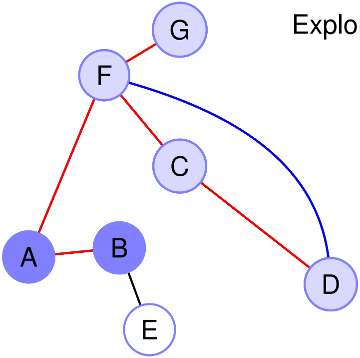

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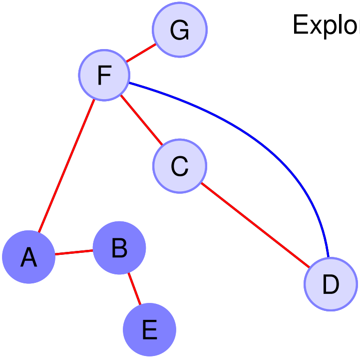

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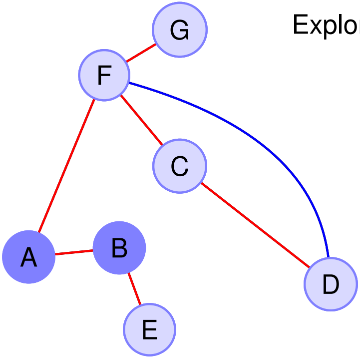

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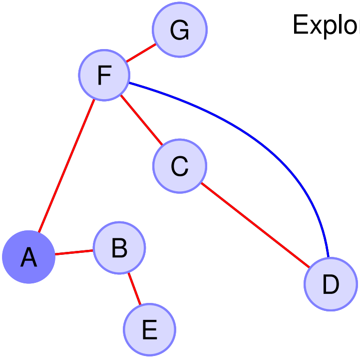

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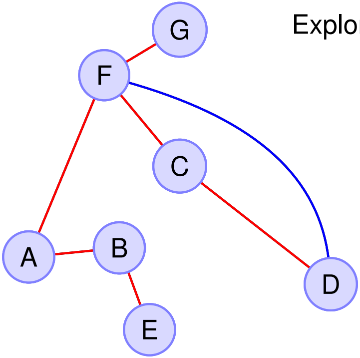

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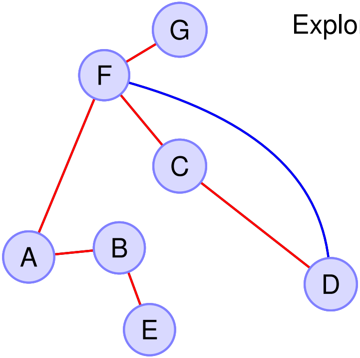

Explore(v):

1. Set visited[v] := true
2. for each edge (v,w) in E
3. if not visited[w]: Explore(w).

Chalk.
Stack is Thread.

Explore builds tree.

Tree and *back* edges.

Correctness.

Explore(v):

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
3.   if not visited[w]: Explore(w)

Property:

All and only nodes reachable from *A* are reached by explore.

Correctness.

Explore(v):

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2. For each edge (v,w) in E
3.   if not visited[w]: Explore(w)

Property:

All and only nodes reachable from *A* are reached by explore.

Only: when *u* visited.

Correctness.

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Property:

All and only nodes reachable from *A* are reached by explore.

Only: when *u* visited.

stack contains nodes in a path from *a* to *u*.

Correctness.

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1. Set visited[v] := **true**.
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Property:

All and only nodes reachable from *A* are reached by explore.

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stack contains nodes in a path from *a* to *u*.

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there is a path to it. Assume: *u* not found.

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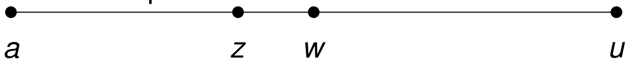

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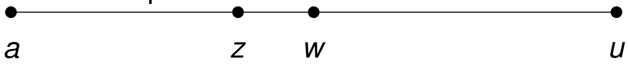

z is explored.

Correctness.

Explore(v):

1. Set visited[v] := **true**.
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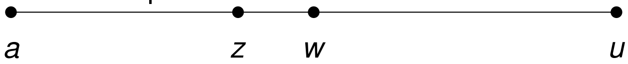


z is explored. *w* is not!

Correctness.

Explore(v):

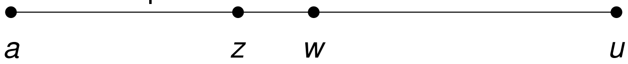
1. Set visited[v] := **true**.
2. For each edge (v,w) in E
3. if not visited[w]: Explore(w)

Property:

All and only nodes reachable from A are reached by explore.

Only: when u visited.
stack contains nodes in a path from a to u.

All: if a node u is reachable.
there is a path to it. Assume: u not found.



z is explored. w is not!
Explore (z) would explore(w), or it was already explored!

Correctness.

Explore(v):

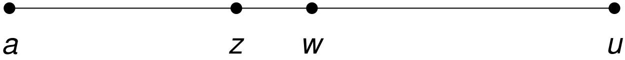
1. Set visited[v] := **true**.
2. For each edge (v,w) in E
3.   if not visited[w]: Explore(w)

Property:

All and only nodes reachable from *A* are reached by explore.

Only: when *u* visited.
  stack contains nodes in a path from *a* to *u*.

All: if a node *u* is reachable.
  there is a path to it. Assume: *u* not found.



z is explored. *w* is not!
Explore (*z*) would explore(*w*), or it was already explored!
Contradiction.

□

Running Time.

Explore(v):

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
3.   if not visited[w]: Explore(w).

Running Time.

Explore(v):

1. Set visited[v] := **true**.
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How to analyse?

Running Time.

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How to analyse?

Let $n = |V|$, and $m = |E|$.

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Let $n = |V|$, and $m = |E|$.

$T(n,m) \leq (d)T(n-1,m) + O(d)$

Running Time.

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How to analyse?

Let $n = |V|$, and $m = |E|$.

$T(n,m) \leq (d)T(n-1,m) + O(d)$ Exponential

?!?!?!

Running Time.

Explore(v):

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How to analyse?

Let $n = |V|$, and $m = |E|$.

$T(n,m) \leq (d)T(n-1,m) + O(d)$ Exponential
?!?!?!

Don't use recurrence!

Running Time.

Explore(v):

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
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Running Time.

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How to analyse?

Let $n = |V|$, and $m = |E|$.

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How to analyse?

Let $n = |V|$, and $m = |E|$.

"Charge work to something."

Running Time.

Explore(v):

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3.   if not visited[w]: Explore(w).

How to analyse?

Let $n = |V|$, and $m = |E|$.

"Charge work to something."

For node x:

Running Time.

Explore(v):

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
3.   if not visited[w]: Explore(w).

How to analyse?

Let $n = |V|$, and $m = |E|$.

"Charge work to something."

For node x:
  Explore once!

Running Time.

Explore(v):

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
3.   if not visited[w]: Explore(w).

How to analyse?

Let $n = |V|$, and $m = |E|$.

"Charge work to something."

For node x:

Explore once!
Process each incident edge.

Running Time.

Explore(v):

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
3.   if not visited[w]: Explore(w).

How to analyse?

Let $n = |V|$, and $m = |E|$.

"Charge work to something."

For node x:

Explore once!
Process each incident edge.

Each edge processed twice.

Running Time.

Explore(v):

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
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How to analyse?

Let $n = |V|$, and $m = |E|$.

"Charge work to something."

For node x:
  Explore once!
  Process each incident edge.

Each edge processed twice.

$O(n)$ - call explore on n nodes.

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Total: $O(n+m)$.

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Process whole graph.

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For each node u,
    visited[u] = false.
For each node u,
    if not visited[u] explore(u)
```

Depth first search.

Process whole graph.

DFS(G)

1: For each node u,
2:    visited[u] = **false**.
3: For each node u,
4:    if not visited[u] **explore(**u**)**

Running time: $O(|V| + |E|)$.

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Process whole graph.

DFS(G)

For each node u,
    visited[u] = **false**.
For each node u,
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Running time: $O(|V| + |E|)$.

Intuitively: tree for each “connected component”.

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Several trees

Depth first search.

Process whole graph.

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DFS and connected components.

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DFS(G):

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4.   ccnum = ccnum+1

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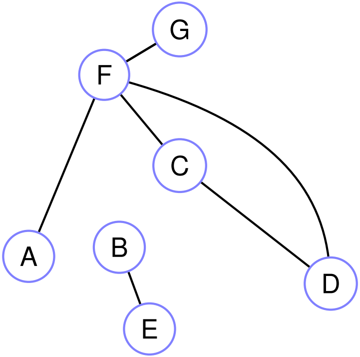
Each node will be labelled with connected component number.
Runtime: $O(|V| + |E|)$.

Connected Components.



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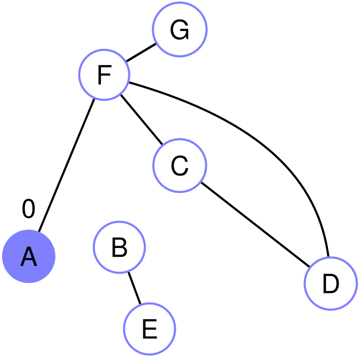

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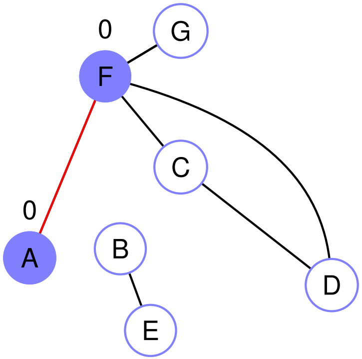

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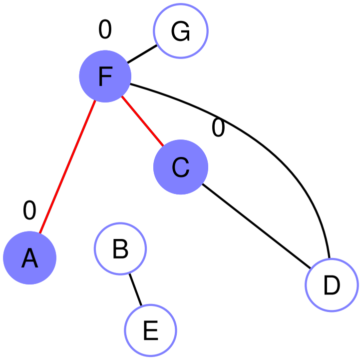

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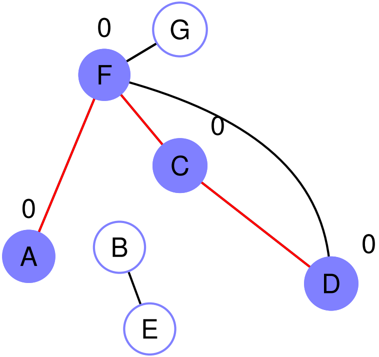

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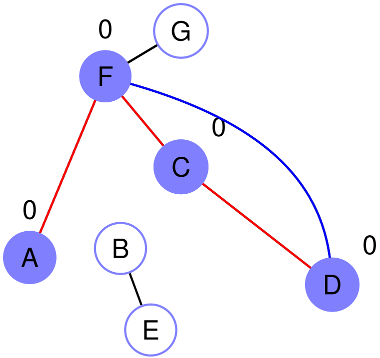

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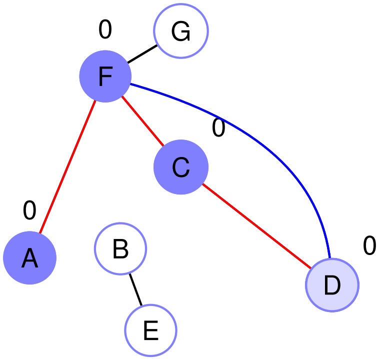

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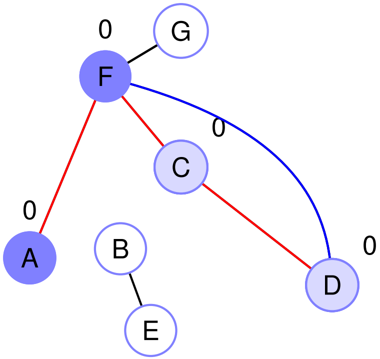

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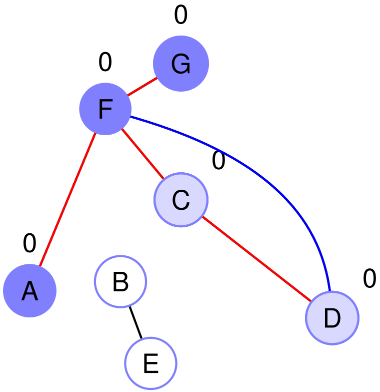

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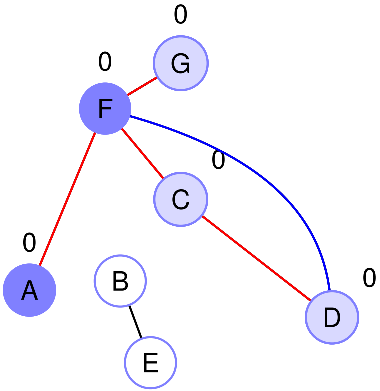

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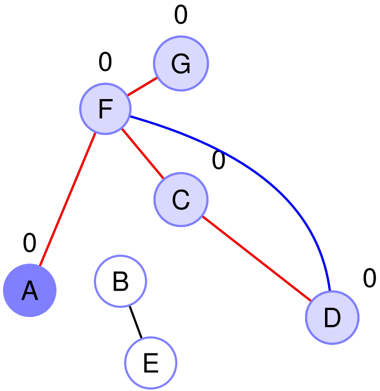

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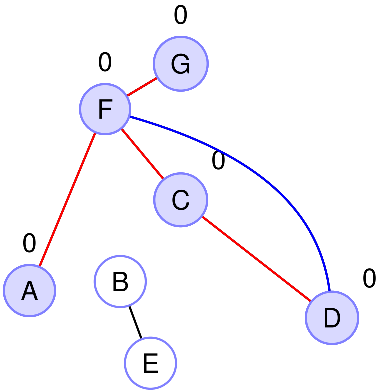

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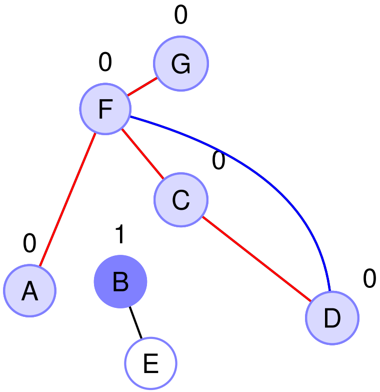

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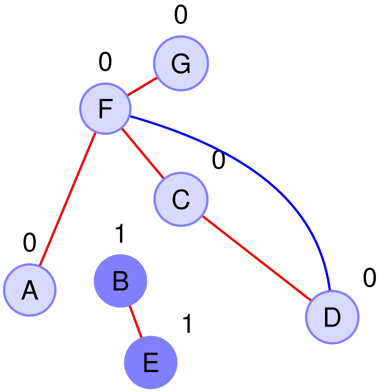

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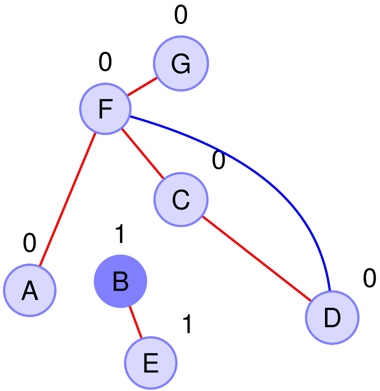

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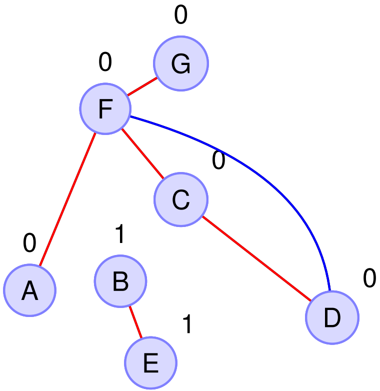

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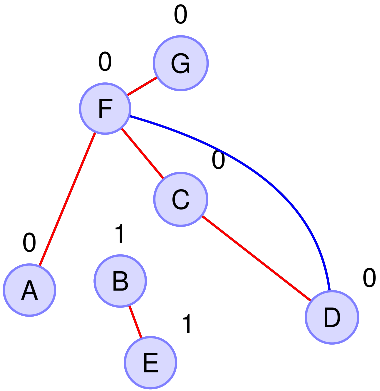

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1. Set pre[v] := clock.
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Clock: goes up to 2 times number of vertices.

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Property:
For any two nodes, u and v, $[pre(u), post(u)]$ and $[pre(v), post(v)]$ are either **disjoint** or **one is contained in other.**

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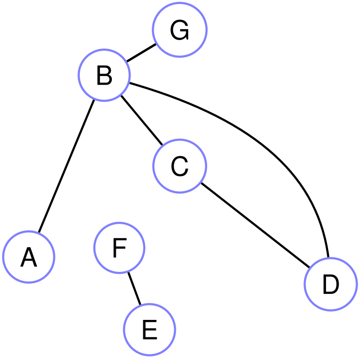
Let’s just watch it work!

Example: Pre/Post numbering.



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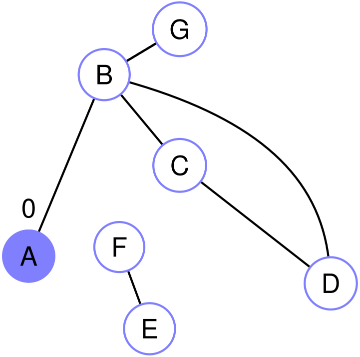

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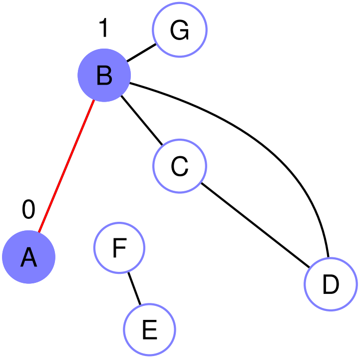

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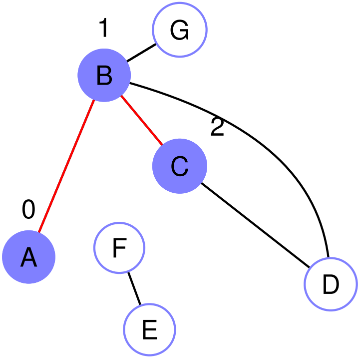

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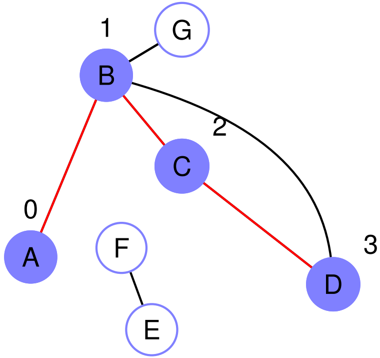

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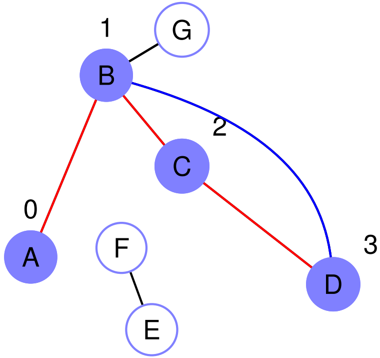

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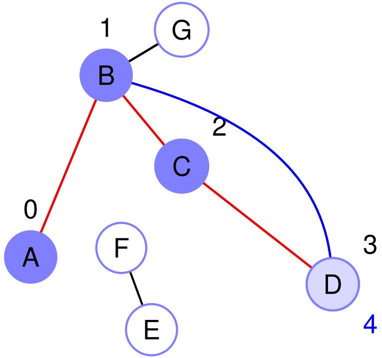

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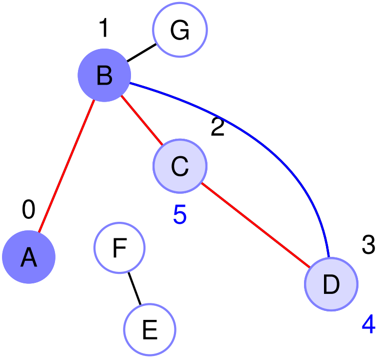

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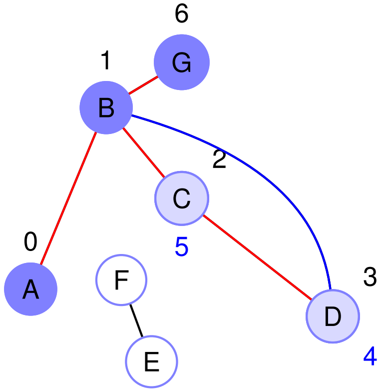

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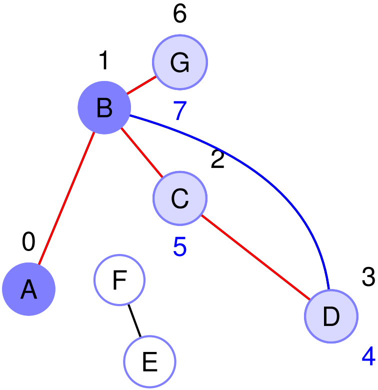

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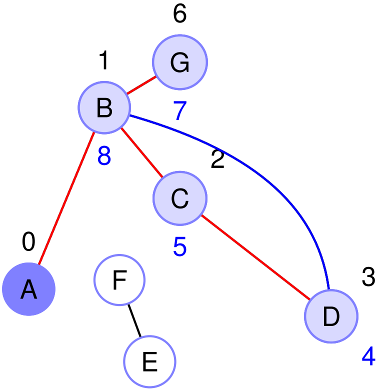

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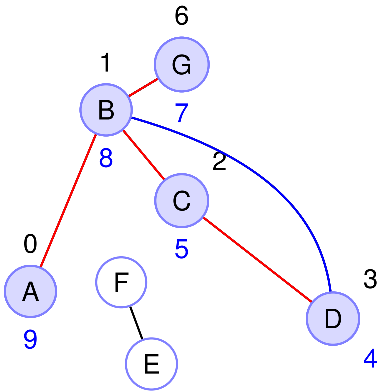

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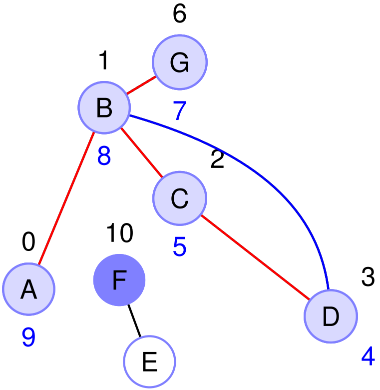

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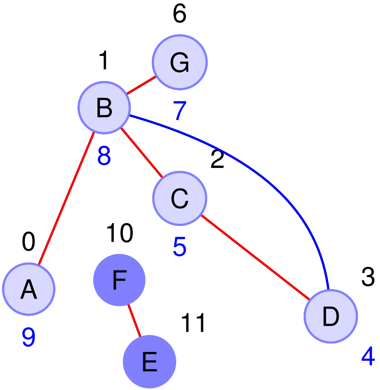

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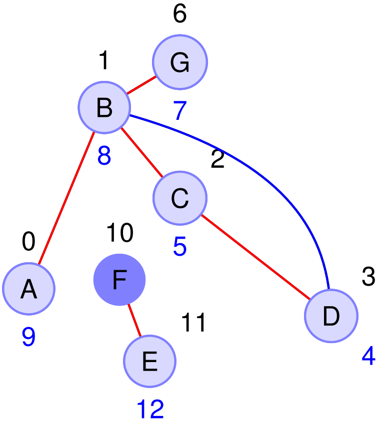

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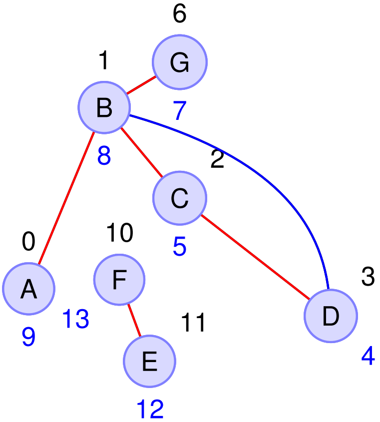

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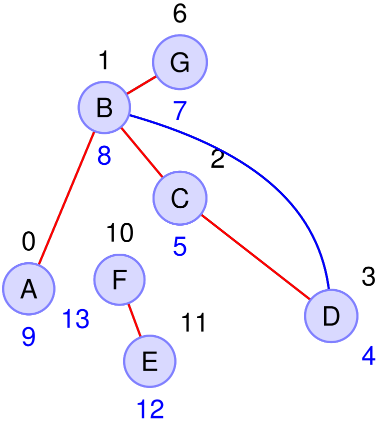

Edge (u, v) is tree edge iff $[pre[v], post[v]] \subset [pre[u], post[u]]$.

u on stack before v.

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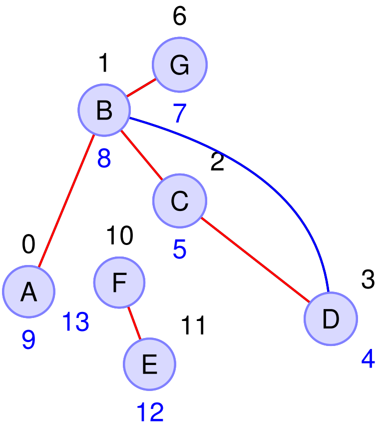

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v on stack before u on stack. Path from v to u! Cycle!

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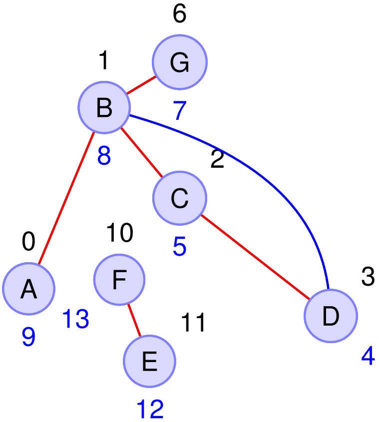

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No edge between u and v if disjoint intervals.

Directed graphs.

$G = (V, E)$

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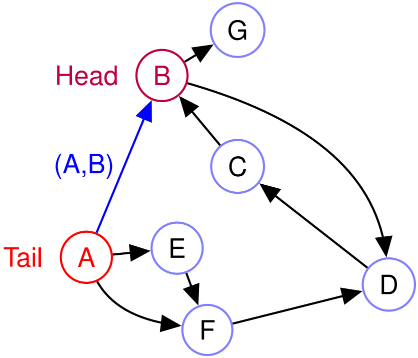
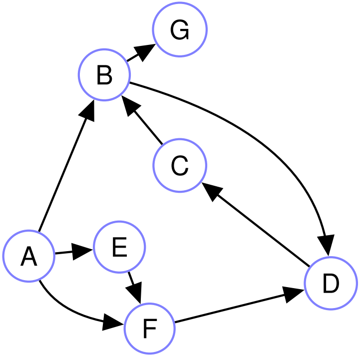


Depth first search: directed.



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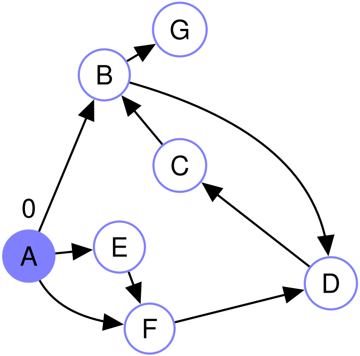

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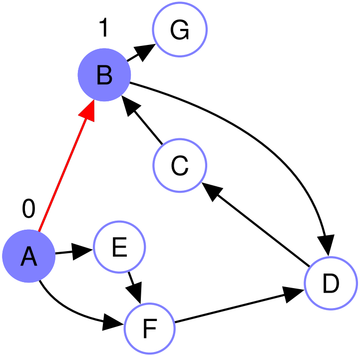

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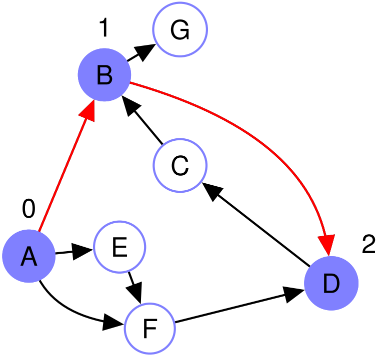

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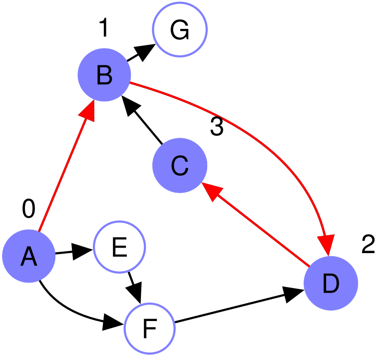

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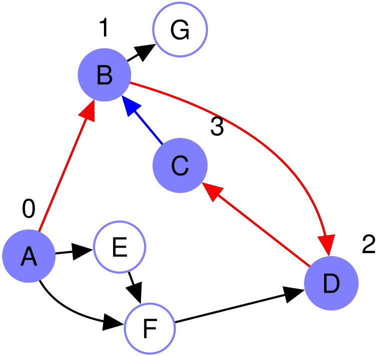

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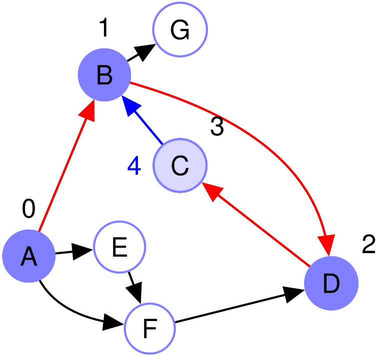

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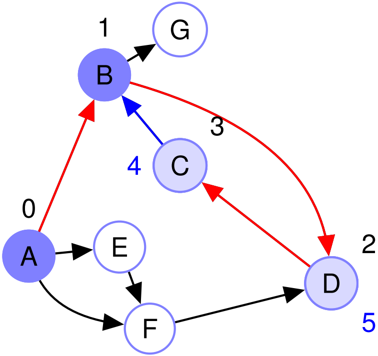

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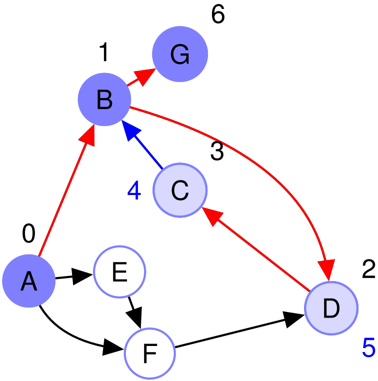

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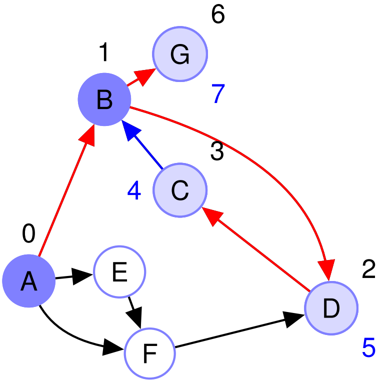

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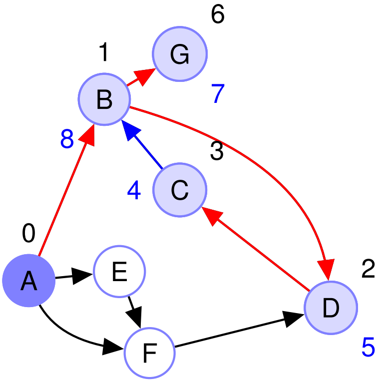

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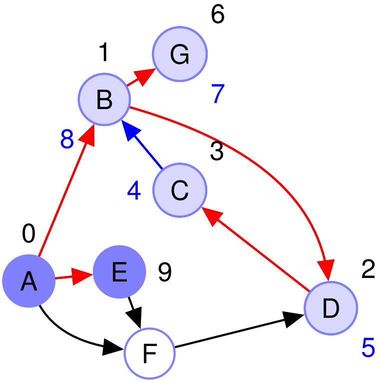

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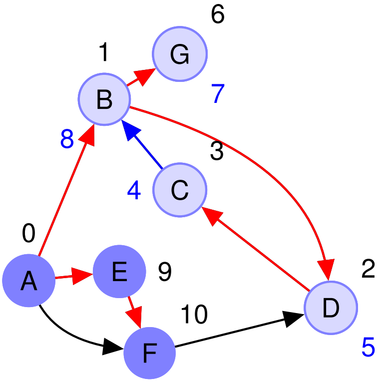

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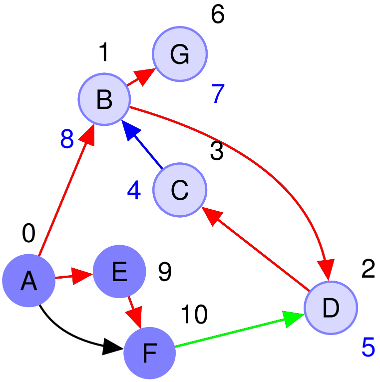

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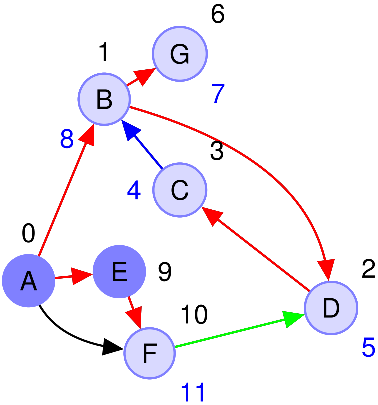

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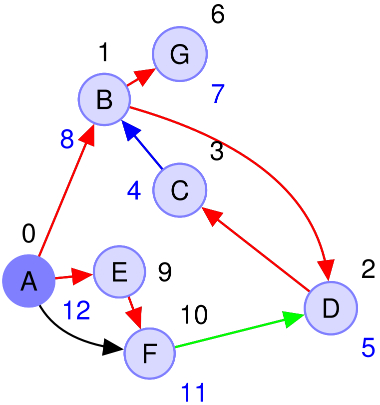

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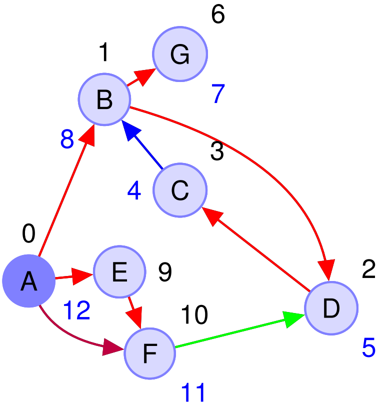

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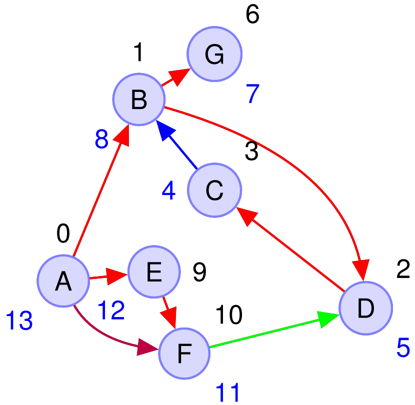

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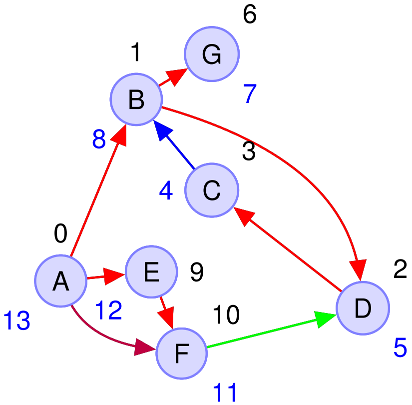

Tree/forward edge (u,v): $int(v) \subset int(u)$. $inv(v) = [pre(v), post(v)]$

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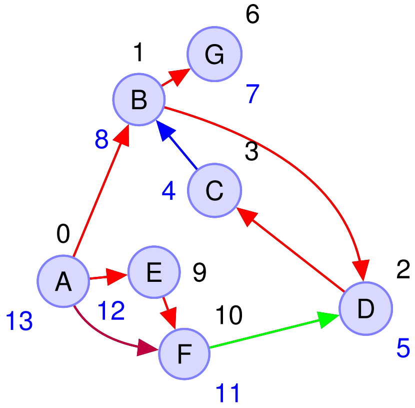

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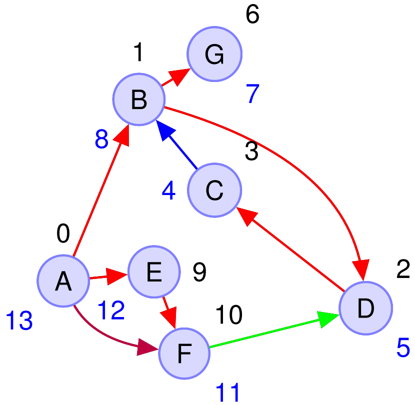

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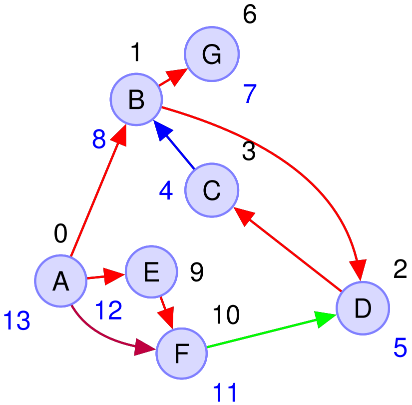

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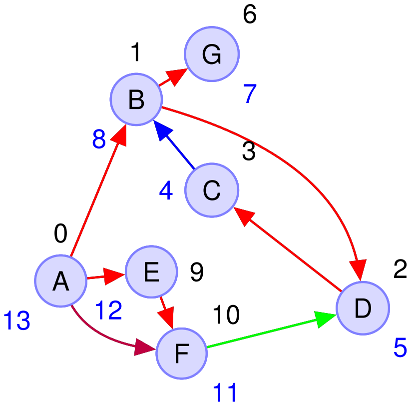

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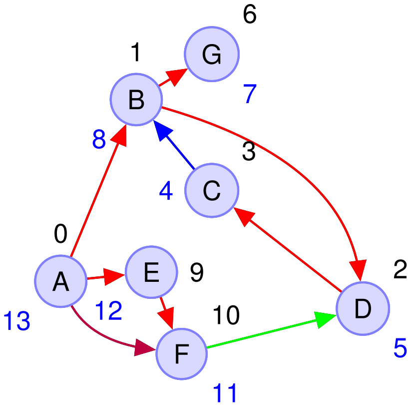

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(F, D): [2,5] before [10,11]

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Linear Time (i.e. $O(|V| + |E|)$)?

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Cycle $\implies$ back edge! $\square$

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$O(|V| + |E|)$ time algorithm for checking if graph is acyclic!

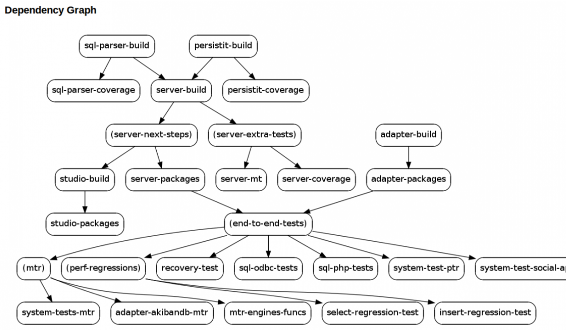
Directed Acyclic Graph

Hello

▼

Goodbye

“Hello” before “Goodbye



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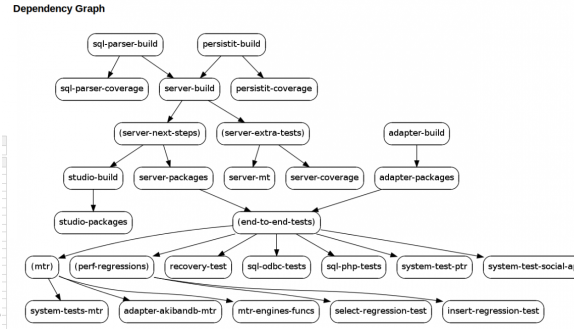

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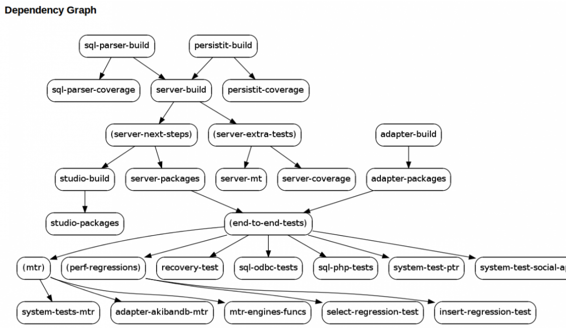

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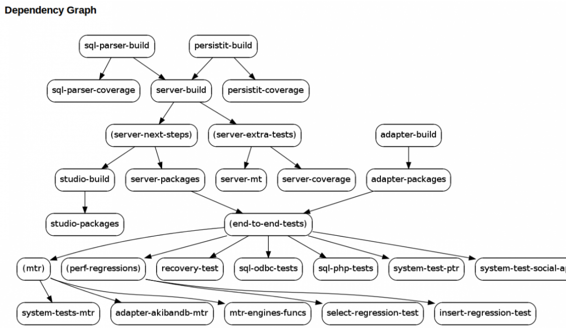

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Really want to find ordering for build!

Linearize.

Topological Sort: For $G = (V, E)$, find ordering of all vertices where each edge goes from earlier vertex to later in acyclic graph.

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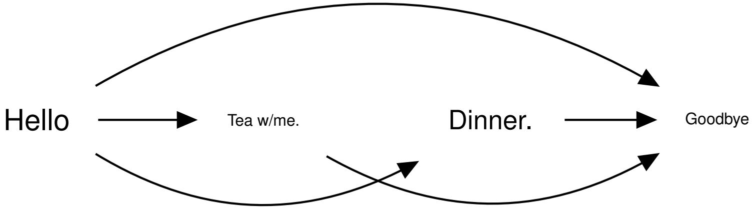

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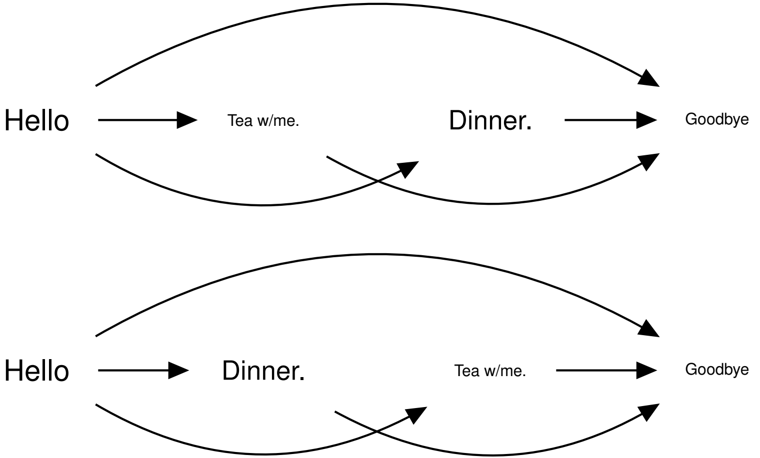
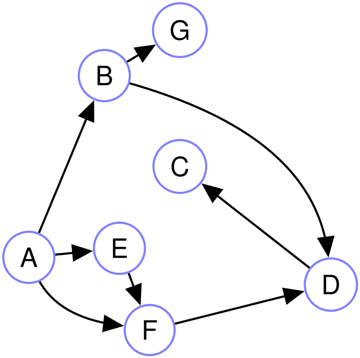


Topological Sort Example.



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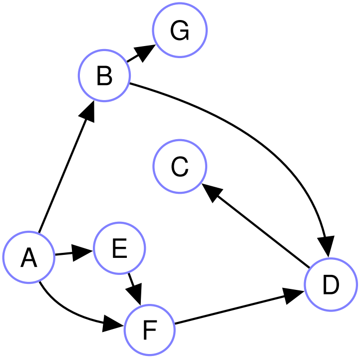

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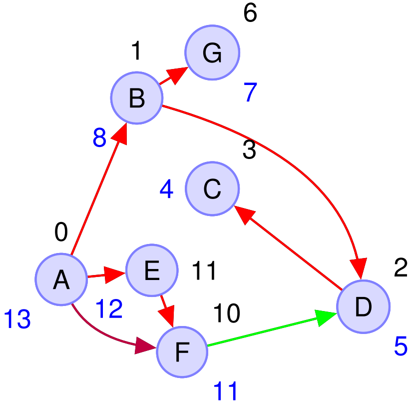

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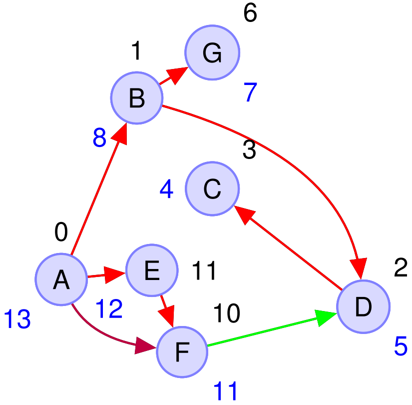

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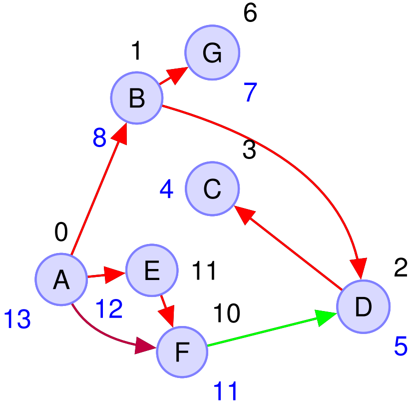

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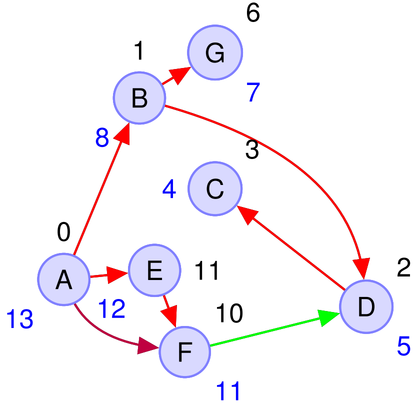

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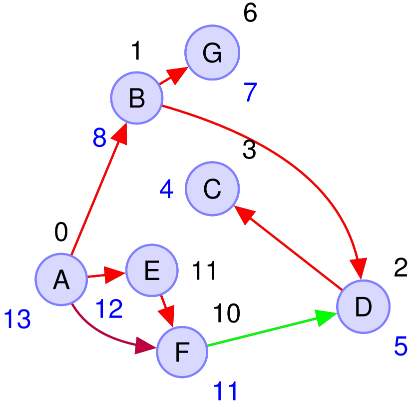

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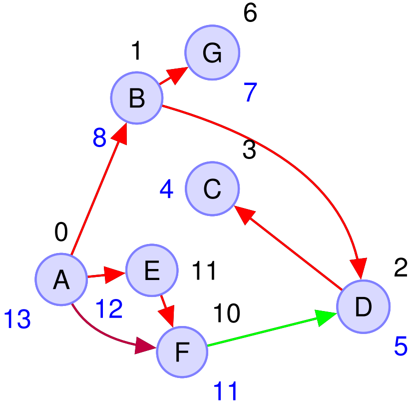

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(A) be first in linearization!

(B) be last in linearization!

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$\square$

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