CS 170: Algorithms

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Static Course Webpage. (cs170.org)

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Will mostly use piazza. (Link.)

CS 170: Algorithms

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Homeworks will be turned on gradescope. Latex ok, scanning handwritten stuff ok.

CS 170: Algorithms

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A puzzle.

Does a list have a cyle?

A puzzle.

Does a list have a cyle?

Access to list is a pointer to the “first element.”

A puzzle.

Does a list have a cyle?

Access to list is a pointer to the “first element.”

Mark first node.

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Mark first node.
While next cell not marked, go to next cell.

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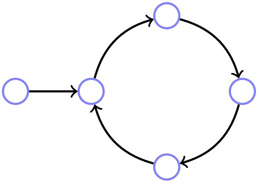

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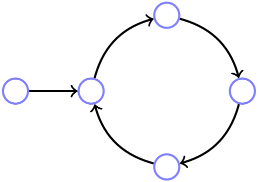

First node not in cycle!

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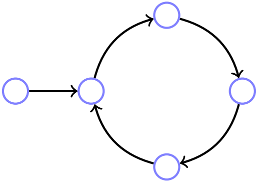

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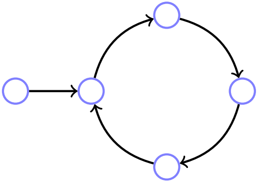

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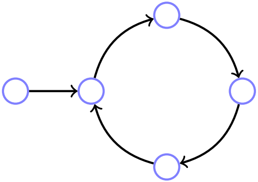

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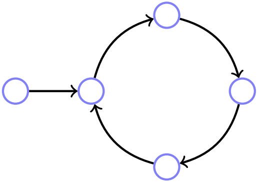

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Problem: starting point is not on cycle?
Construct example.

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Two ptrs:

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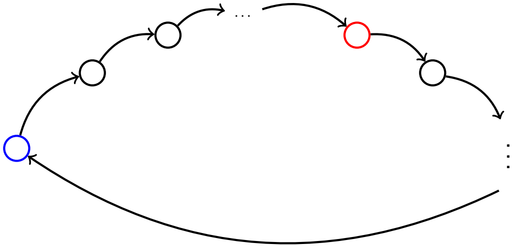

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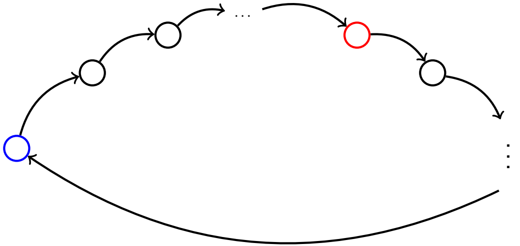

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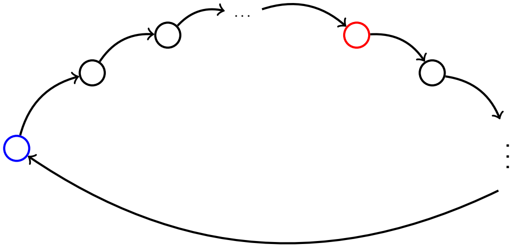

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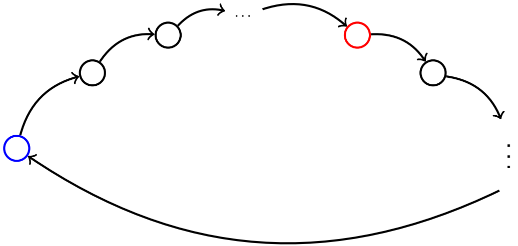

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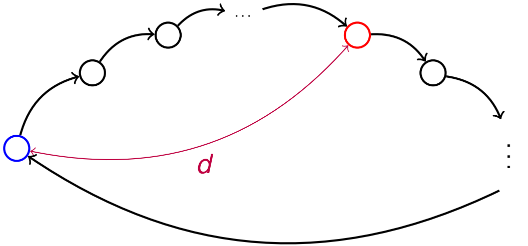

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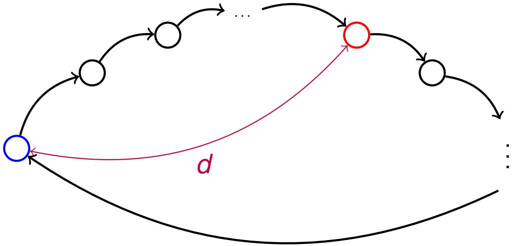

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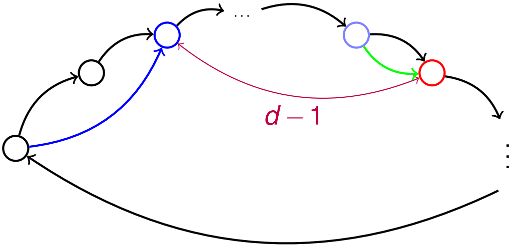

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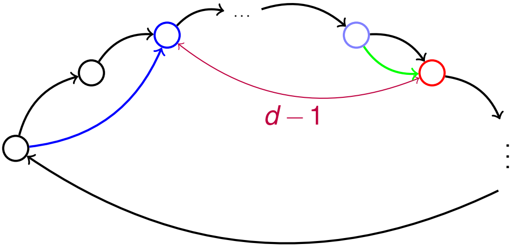

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Runtime: n steps to cycle

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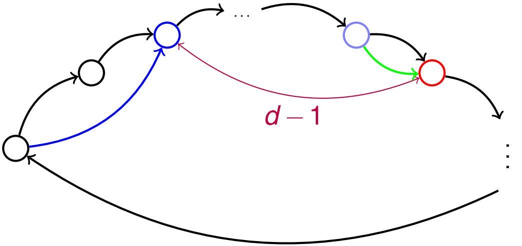

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Runtime: n steps to cycle n steps to catch up.

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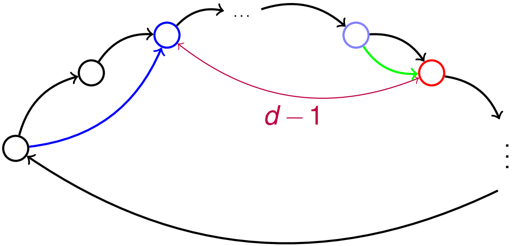

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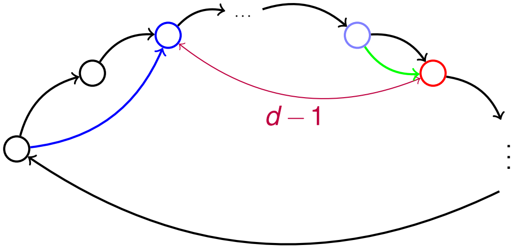

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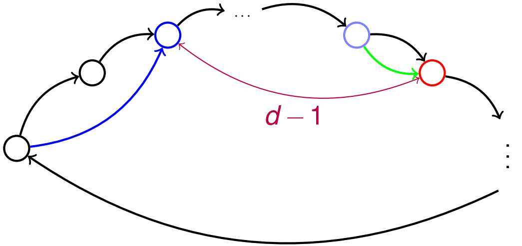

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Runtime: n steps to cycle n steps to catch up. $O(n)$
Additional storage: two pointers. $O(1)$.

Puzzles ..

Solutions

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..are Algorithms...

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which...

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Is this a useful process?

Algorithms for the Human Genome Project

Reconstruct DNA...

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ACTGAAACTGAGTAGATA....

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Read first, then next, then next, ...3.1 billion times...
.. slow... error prone...

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Page Rank.

Problem: What is good on the web?

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Website that pays search engine most?

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New Model for user: google.

Algorithms...

Driving Directions

Algorithms...

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Airline Scheduling

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.
.

Calculating: 300 BC through Middle Ages in Europe.

I – one

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five

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I – one
V – five
X – ten

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten
C – one hundred

Calculating: 300 BC through Middle Ages in Europe.

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X – ten
C – one hundred
D – five hundred

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I – one
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X – ten
C – one hundred
D – five hundred
M – a thousand

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Calculating: 300 BC through Middle Ages in Europe.

I – one
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VIII – eight

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DCLXXI – five hundred plus a hundred plus fifty plus ten plus ten..

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Add them?

1448 + 671 =

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Multiply roman numbers?

Modern system.

From India, via Al Khwarizmi.

Modern system.

From India, via Al Khwarizmi.

He also described recipes for adding, multiplying, solving quadratics, computing digits of π..

Modern system.

From India, via Al Khwarizmi.

He also described recipes for adding, multiplying, solving quadratics, computing digits of π..

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The input representation for modern computers and communication.

Writing to propagating..

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Italian mathematician (1170-1250) who traveled to learn the Hindu-Arab math.

Fibonacci numbers.

$F_0 = 0$, $F_1 = 1$.

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Fibonacci algorithm and numbers!

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Can we do better?

Better Algorithm.

Better Algorithm.

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    a = [0,1]
    for i in xrange(2,n+1):
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Let's try it.

From demo: Size matters.

How many bits in the representation of F_n?

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n additions.

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At most $O(n^2)$.

Demo: polynomial.

Doubling size, made fast fib grow by factor of roughly four.

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Not true for exponential algorithms. Squares runtime!

Asymptotic Analysis.

Used $O(n)$ for number of additions, rather than $n-2$.

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For $2^{.694n}$, doubling n, squares run time.

For $O(n^2)$, doubling n, multiplies run time by four.

Refreshing Asymptotic Notation.

Ignore constant factors.

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Formally, for positive functions g, f from integers to reals,
$g(n) = O(f(n))$, if there is a constant c where $g(n) \leq cf(n)$.

More asymptotic notation.

Ω notation.

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$g(n) = \Theta(f(n))$ if $g(n) = O(f(n))$ and $g(n) = \Omega(f(n))$.

Al Khwarizmi: Arithmetic.

Addition: $O(n)$

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$$
\begin{array}{cccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
+ & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
\end{array}
$$

Al Khwarizmi: Arithmetic.

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\begin{array}{cccccccccc}
 & & & & & 1 & 1 & 1 & 1 & \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
+ & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
 & & & & & & 4 & 4 & 8 & 0
\end{array}
$$

Al Khwarizmi: Arithmetic.

Addition: $O(n)$

$$
\begin{array}{cccccccccc}
 & & & & 0 & 1 & 1 & 1 & 1 & \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
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\hline
 & & & & & 9 & 4 & 4 & 8 & 0
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	0	4	4	6	9	4	4	8	0

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1	0	4	4	6	9	4	4	8	0

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Time: $O(n)$

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1 & 0 & 4 & 4 & 6 & 9 & 4 & 4 & 8 & 0
\end{array}$$

Time: $O(n)$

Can we do better?

Al Khwarizmi: Arithmetic.

Addition: $O(n)$

1	0	0	0	0	1	1	1	1	
	1	2	3	4	5	6	7	8	9
+	9	2	1	2	3	7	6	9	1
1	0	4	4	6	9	4	4	8	0

Time: $O(n)$

Can we do better?

Need to look at the numbers to add them...

Al Khwarizmi: Arithmetic.

Addition: $O(n)$

$$
\begin{array}{cccccccccc}
1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
+ & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
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1 & 0 & 4 & 4 & 6 & 9 & 4 & 4 & 8 & 0
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$$

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More Al Khwarizmi's: algorithms.

Addition: $O(n)$

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Addition: $O(n)$

Multiplication:

$$
\begin{array}{rccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\times & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
\end{array}
$$

More Al Khwarizmi's: algorithms.

Addition: $O(n)$

Multiplication:

$$
\begin{array}{rccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\times & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
$$

More Al Khwarizmi's: algorithms.

Addition: $O(n)$

Multiplication:

$$
\begin{array}{rrrrrrrrrrr}
 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 & \times & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
9 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 &
\end{array}
$$

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Addition: $O(n)$

Multiplication:

$$
\begin{array}{ccccccccccc}
 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 & \times & 9 & 2 & 1 & 2 & 3 & 7 & 6 & 9 & 1 \\
\hline
 & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
9 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & \\
. & . & . & . & . & . & . & . & . & . & .
\end{array}
$$

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Addition: $O(n)$

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\hline
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 & 9 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & \\
 & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & .
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$$

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 & & 9 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & \\
 & & . & . & . & . & . & . & . & . & . & . & . \\
 & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & .
\end{array}
\qquad \updownarrow\, n
$$

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 & & 9 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & \\
 & & . & . & . & . & . & . & . & . & . & . & . \\
 & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & .
\end{array}
\qquad \updownarrow n
$$

Time: $O(n^2)$

Multiplication

Multiplication: $O(n^2)$.

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Is the best possible?

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Every digit in x must multiply every digit in y at least once!

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(a) Yes.

(b) No.

Multiplication

Multiplication: $O(n^2)$.

Is the best possible?

Every digit in x must multiply every digit in y at least once!
$\Theta(n^2)$ such pairs.

Is this the best possible?

(a) Yes.

(b) No.

No.

Multiplication

Multiplication: $O(n^2)$.

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Every digit in x must multiply every digit in y at least once!
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No. We can do better!

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What ?!?!

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Quick Thoughts.

Big (historic) idea: representation as digits or bits.

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Complexity or runtimes in terms of size of representation.

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Asymptotic analysis.