CS 170
Efficient Algorithms and Intractable Problems

Lecture 10
Minimum Spanning Trees

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EECS, UC Berkeley

Announcements

Midterm 1 next Tuesday Feb 25
→ No class on Tuesday!
→ No discussion sections on Tuesday!
→ Midterm 1 Review Sessions: 9-11 Friday @Soda 306
→ Also there are more OHs
→ Scope: Up to and including today's lecture!

HW4 is due this Saturday.
HW 5 is optional and not for grade.
→ Posted with solutions, so review the solutions!

Last Lecture: Minimum Spanning Trees

Minimum Spanning Tree (MST) Problem:
Input: a weighted graph $G = (V, E)$ with non-negative weights.
Output: A tree $T \subseteq E$ connecting all the vertices of the graph with **smallest cost** $\sum_{e \in T} w_e$

Recap: We prove that any algorithm that fits the following meta algorithm correctly returns an MST.

"Cut Property":
If X is a subset of an MST and has no edges from S to $V \setminus S$, then $X \cup \{e\}$ is also a subset of an MST.

Meta Algorithm for MST

$X = \{\}$
Repeat until $|X| = |V| - 1$
  Pick $S \subset V$, s.t. X has no edges from S to $V \setminus S$
  $e \leftarrow$ lightest weight edge from S to $V \setminus S$
  $X \leftarrow X \cup \{e\}$

Today

Two algorithms for MST.

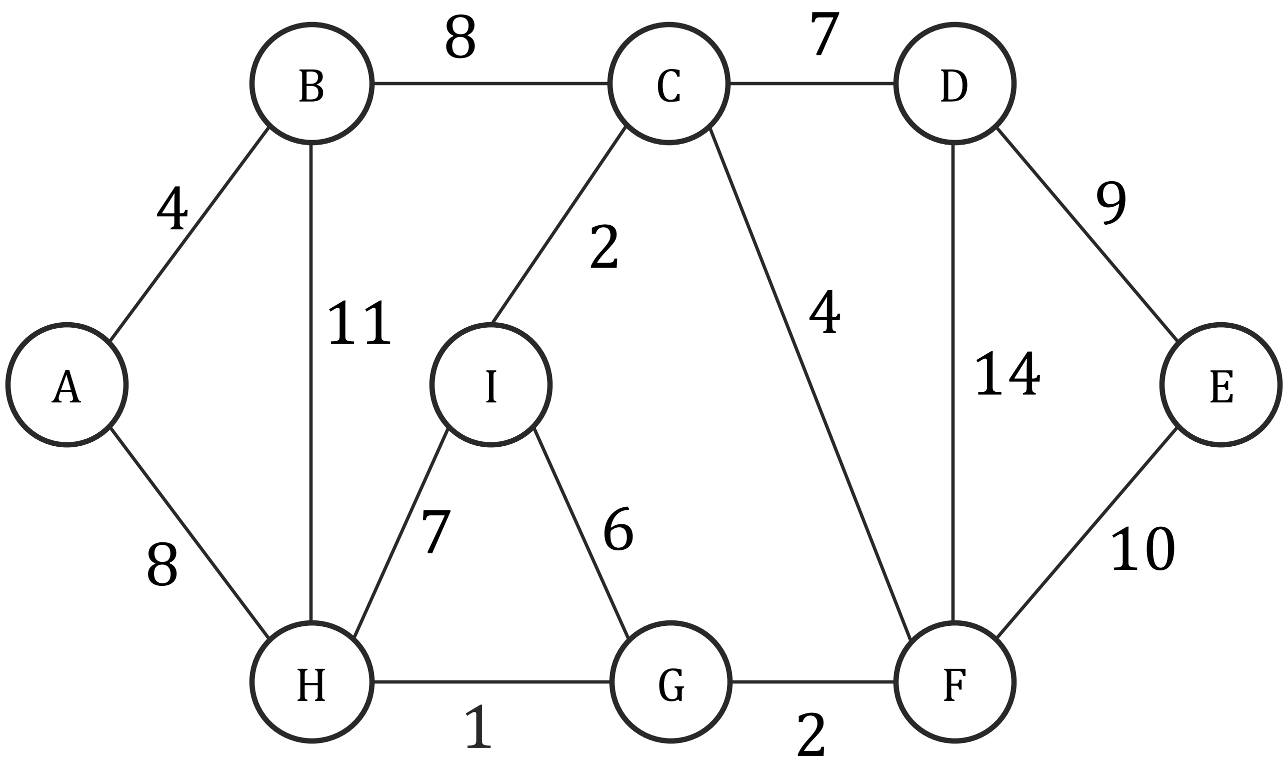
They fit the meta-algorithm recipe!

Based on different choice of cuts.

Kruskal's Algorithm

Instead of explicitly defining $S, V \setminus S$, Kruskal's algorithm picks $e = (u, v)$ directly and ensures that (u, v) is the lightest edge crossing **some cut**.

Which cut? $S, V \setminus S$ correspond to **connected components for u and v**.



Kruskal(G = (V,E)):

X = {}

for $e \in E$ in **increasing order of weight**

If adding e to X doesn't create a cycle

$X \leftarrow X \cup \{e\}$.

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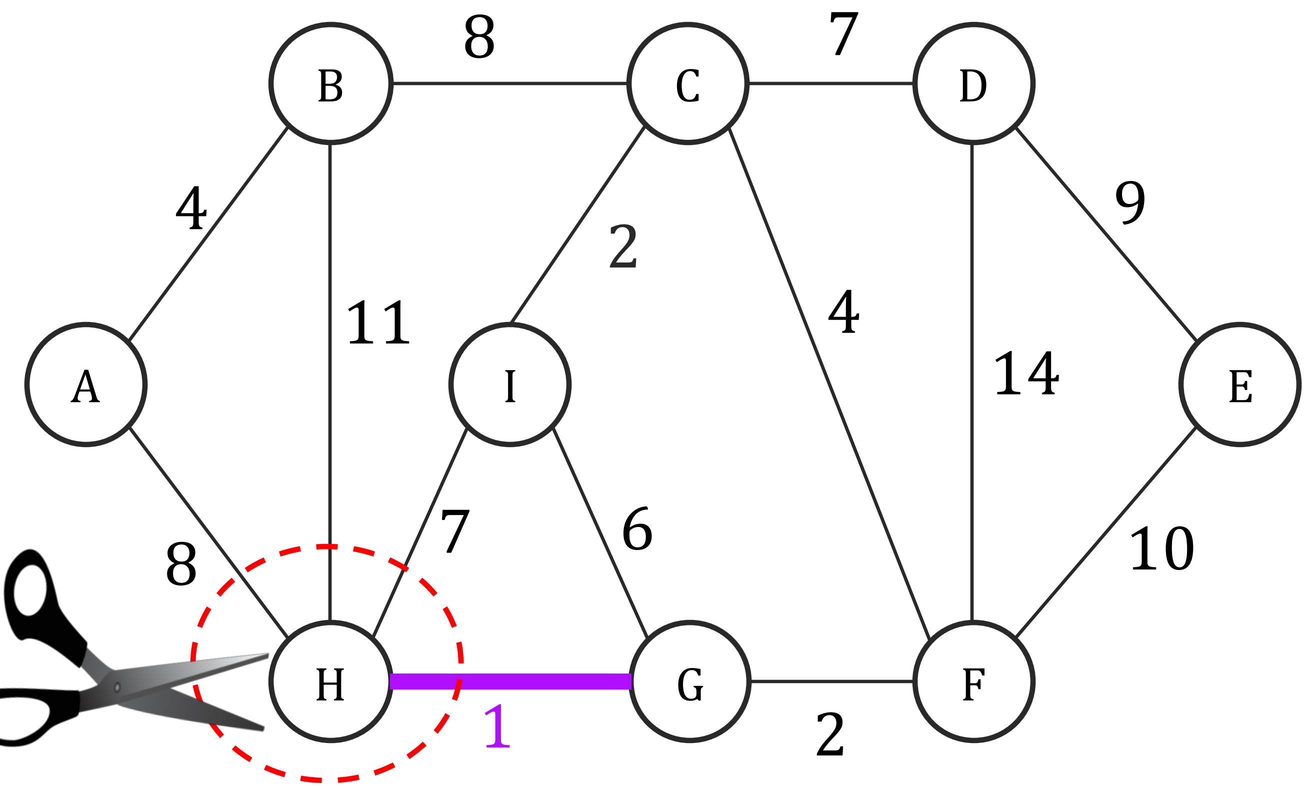

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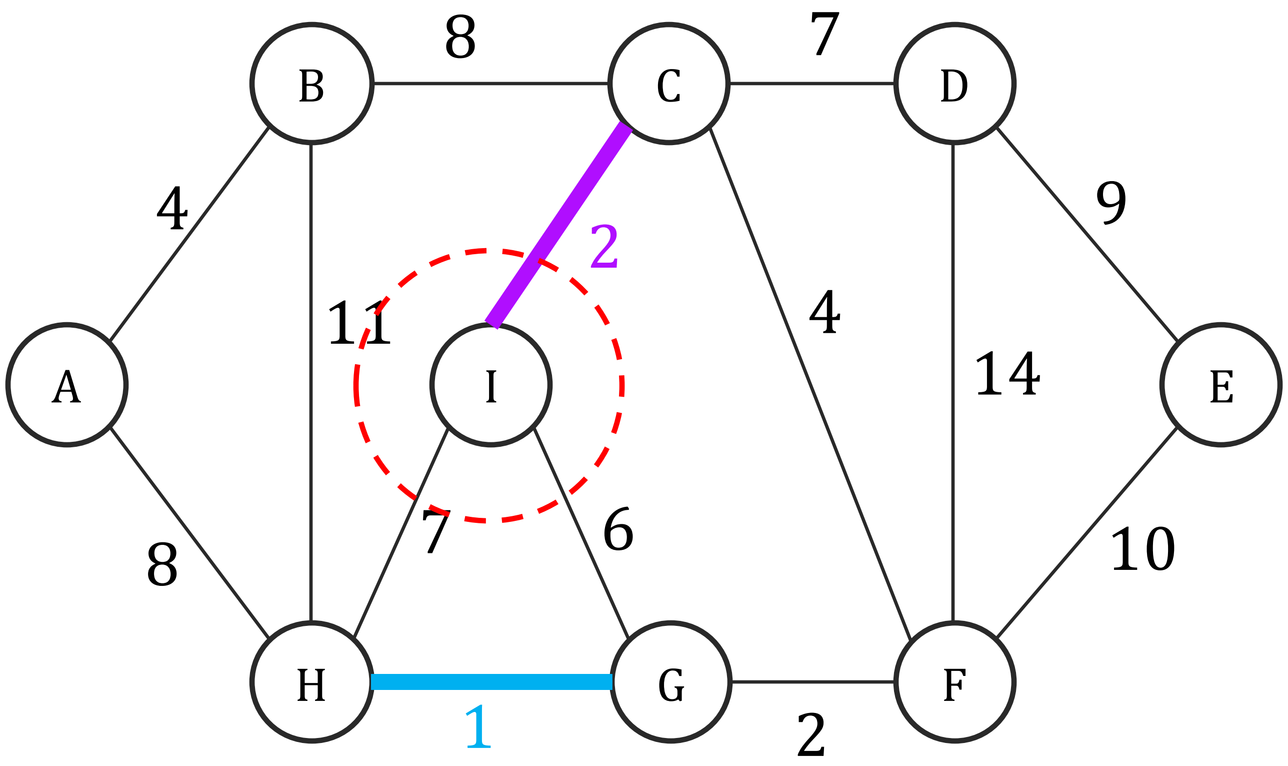

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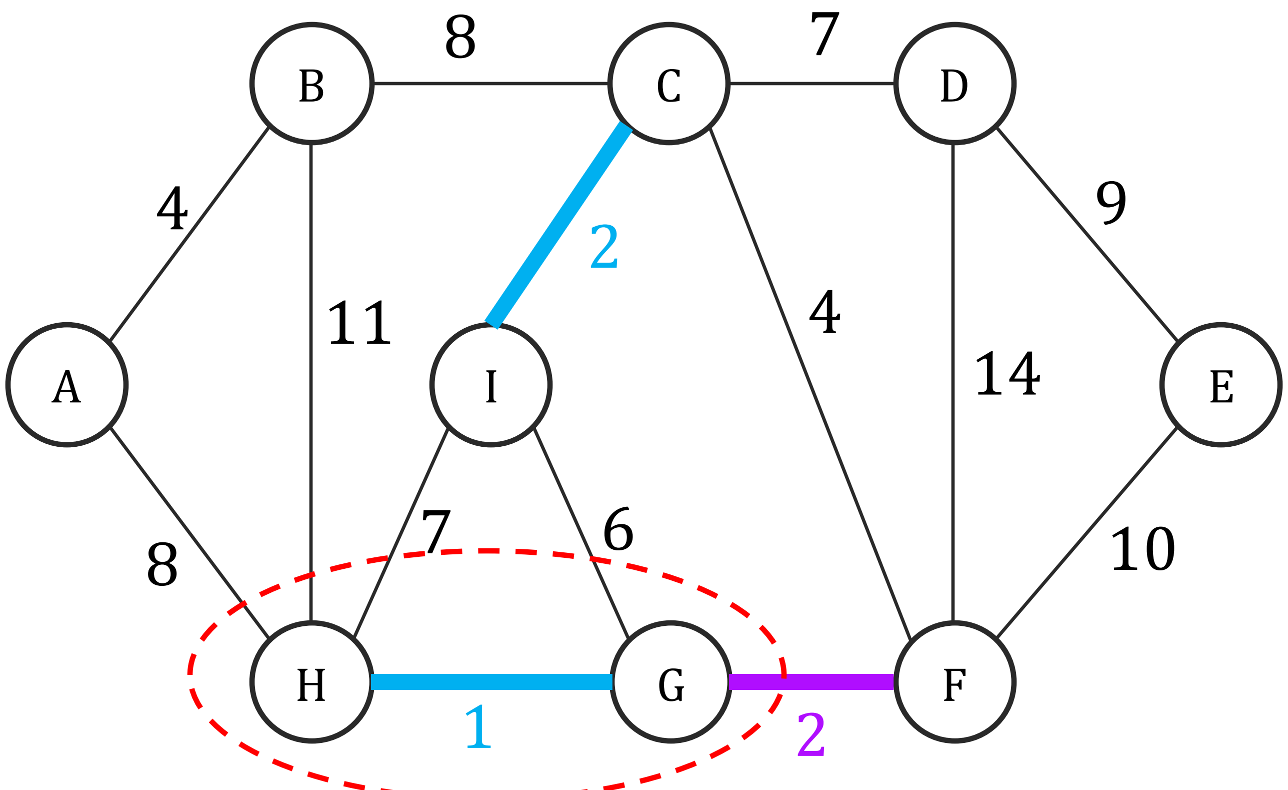

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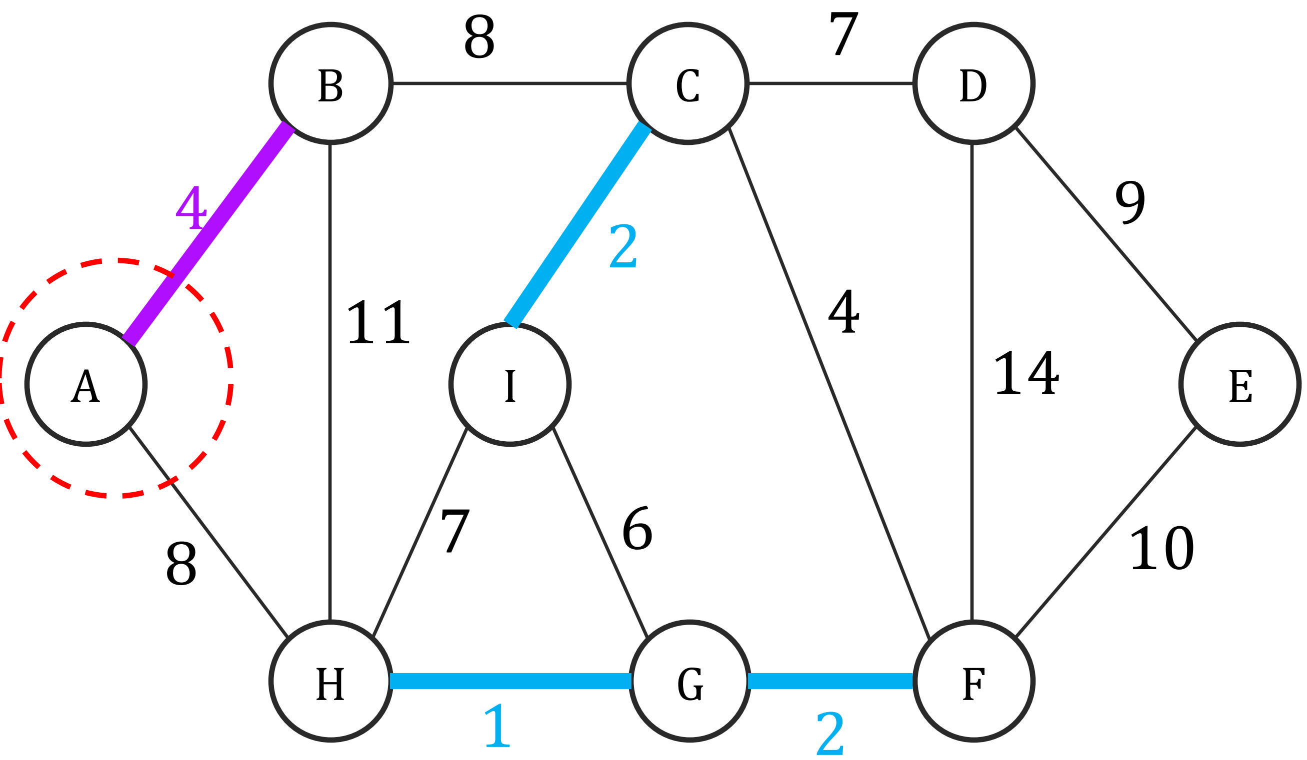

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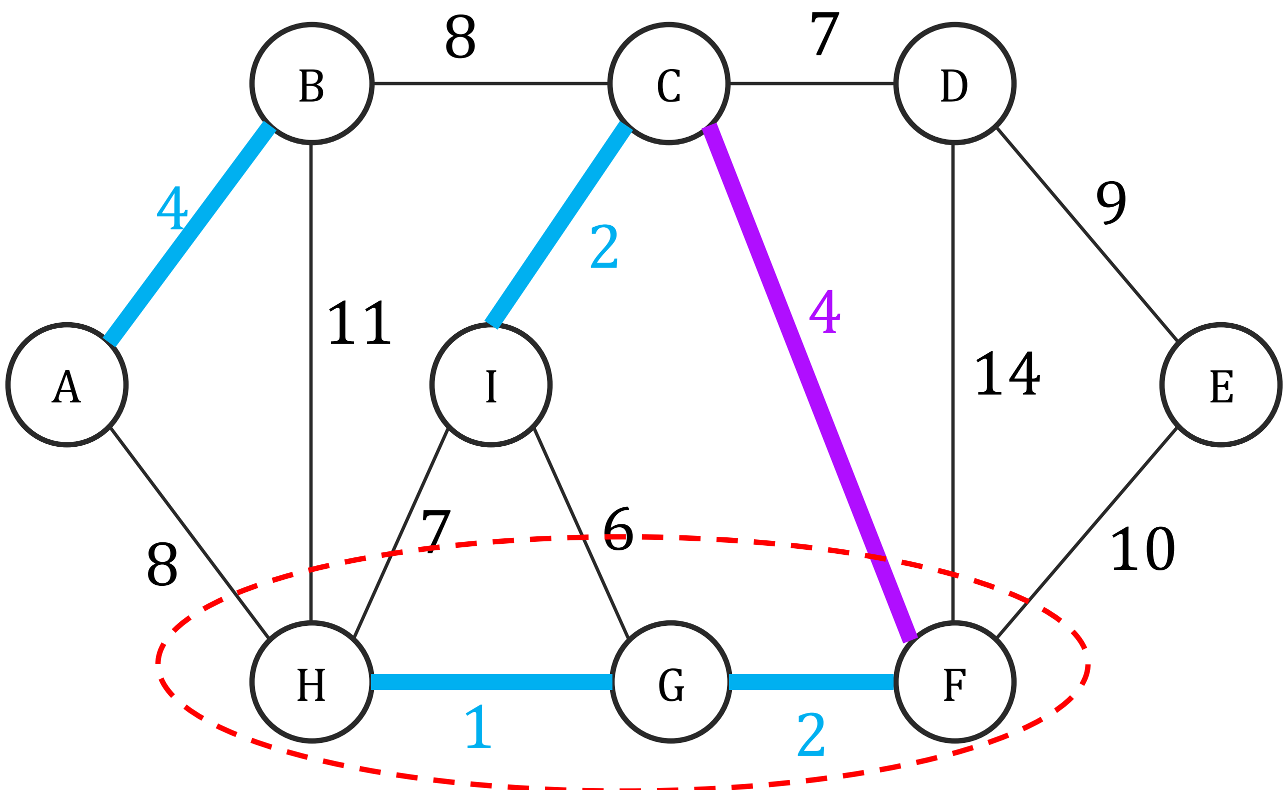

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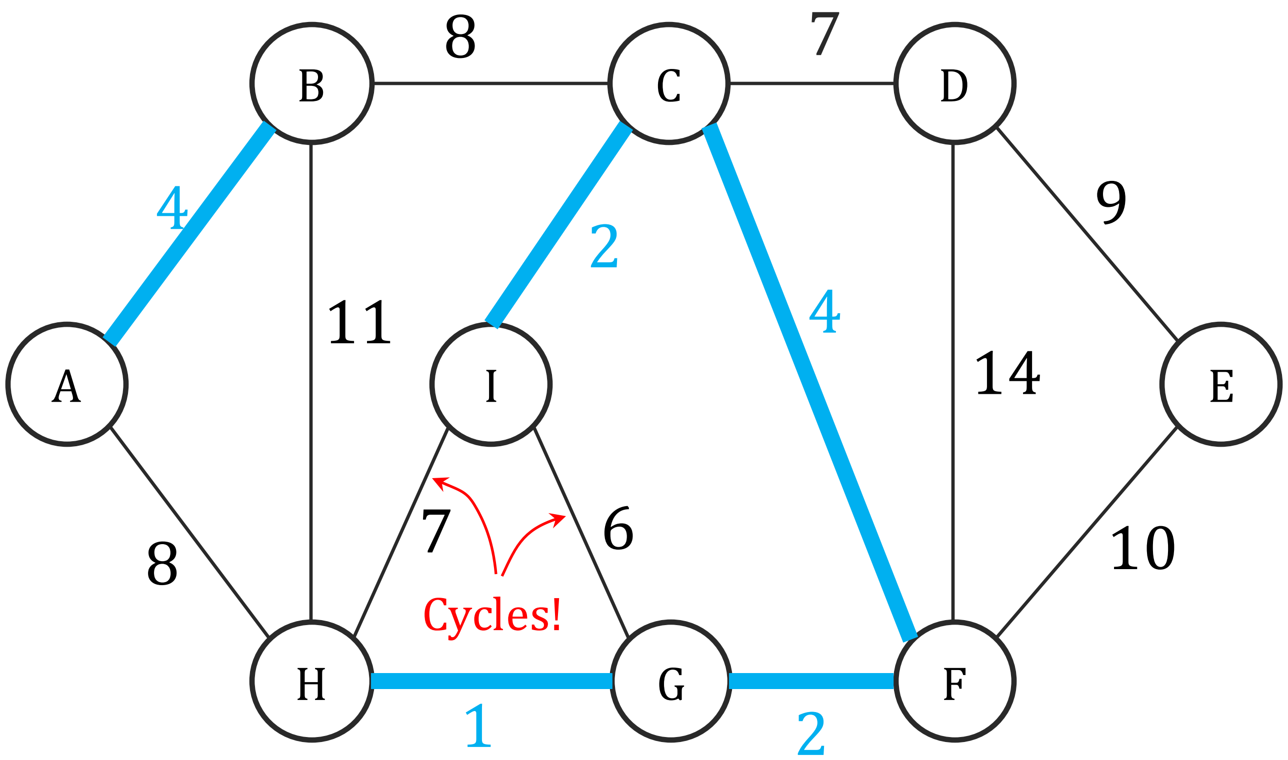

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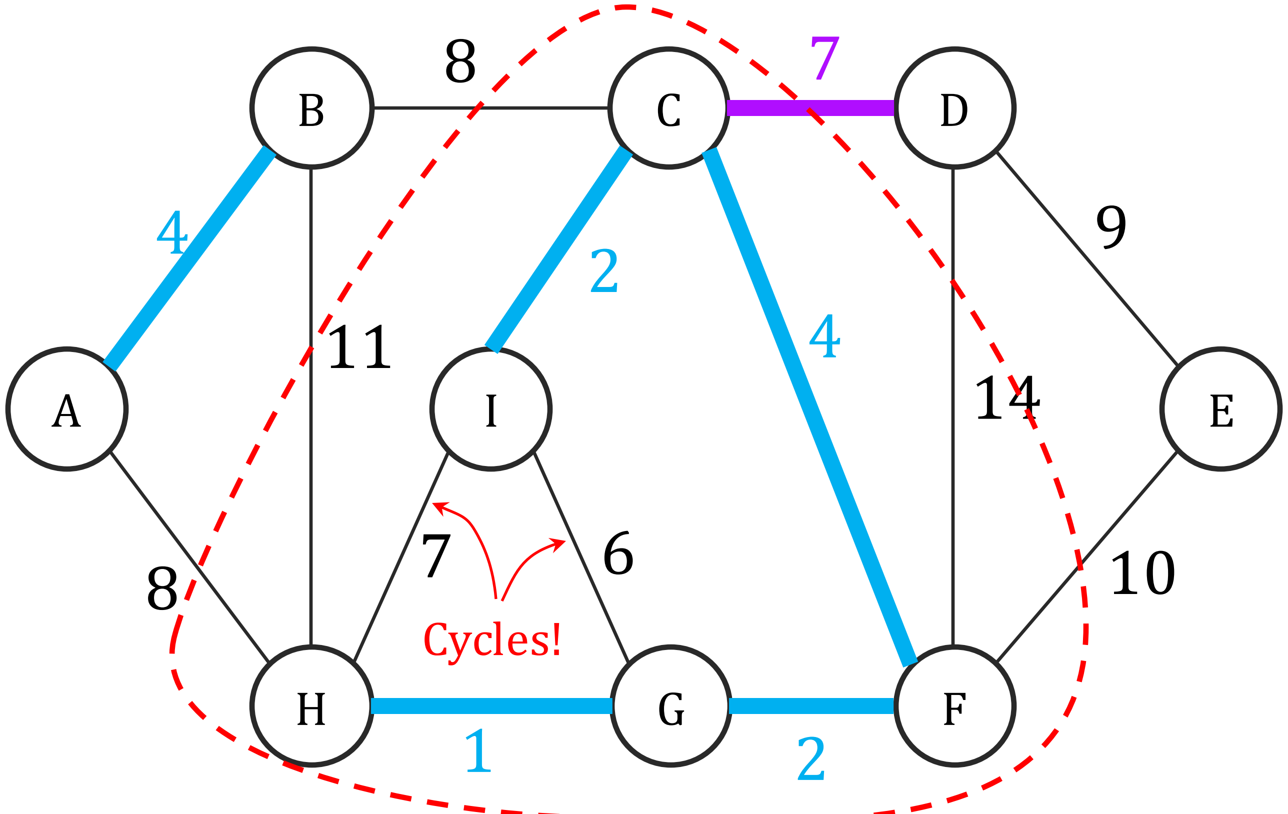

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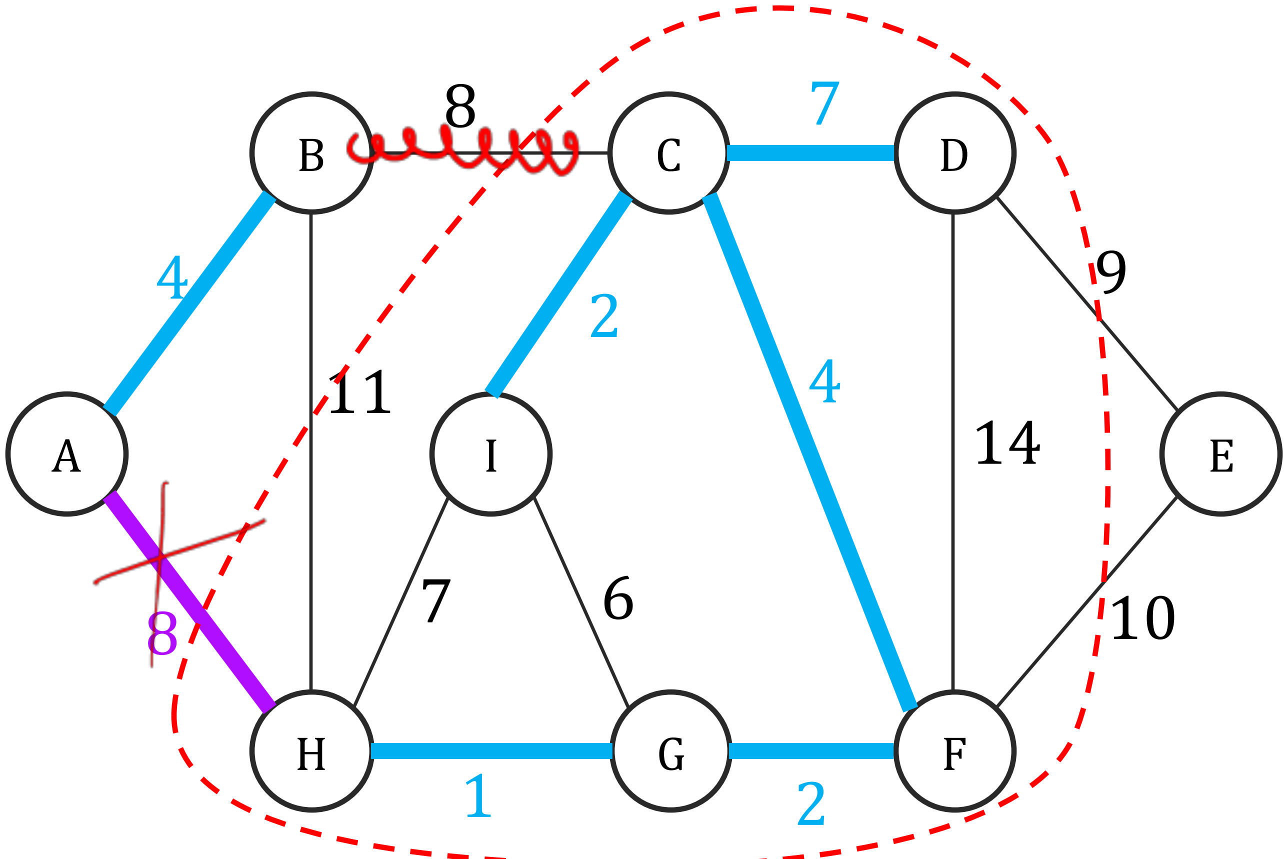

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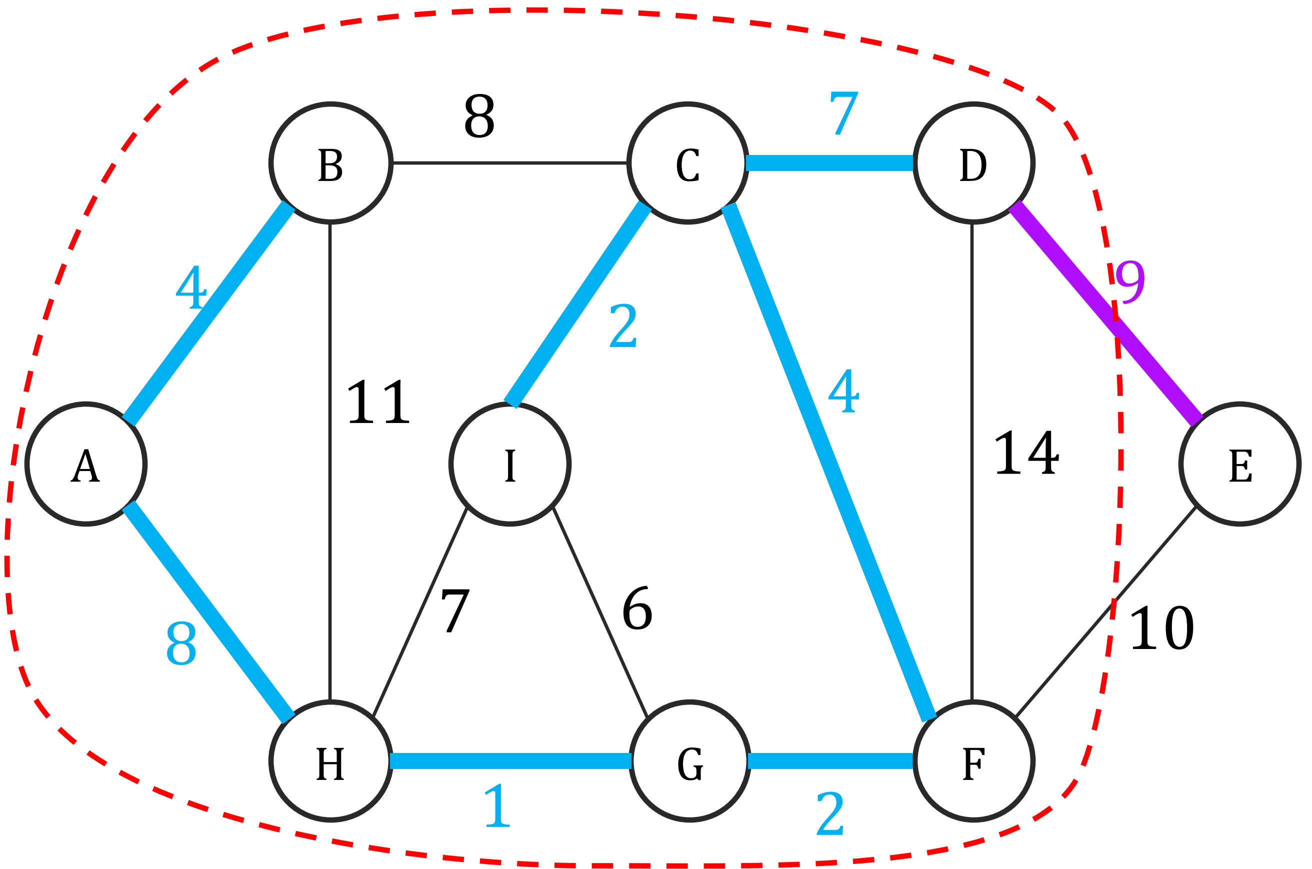

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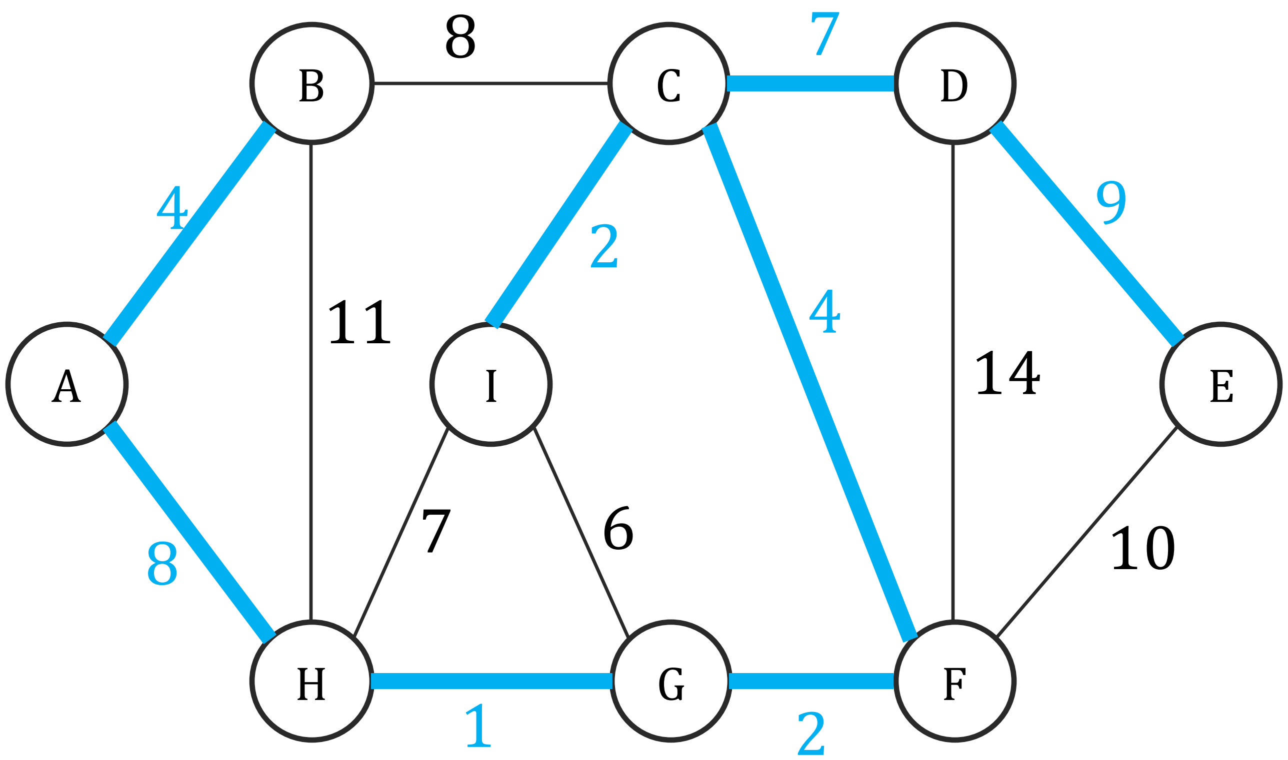

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Kruskal's Correctness

Does Kruskal return a minimum spanning tree?

- Since $X \cup \{(u, v)\}$ doesn't have a cycle, u and v belong to two different connected components of X.
- Let $S \leftarrow$ Connected component including u
- So (u, v) is the lightest edge from S to $V \setminus S$.

→Kruskal fits the meta algorithm description, so it find an MST.

Meta Algorithm for MST

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Kruskal's Runtime and Union-Find

How do we quickly check if $X \cup \{(u, v)\}$ has a cycle?

→ We need to check if u's connected component in X = v's connected component in X

Union-FIND: A data-structure for **disjoint sets**

- makeSet(u): create a set from element u. Takes $O(1)$
- find(u): return the set that includes element u. Takes $O(\log(n))$
- union(u, v): Merge two sets containing u and v. Takes $O(\log(n))$

```
Fast-Kruskal(G = (V,E)):
  for v ∈ V, makeSet(v)
  for edges (u, v) ∈ E in increasing order of weight
        If find(v) ≠ find(u)
              X ← X ∪ {(u, v)}
              union(u, v)
return X
```

Runtime of Kruskal's Algorithm

Sorting m edges: $O(m\log(m)) = O(m\log(n))$. Since $m \leq n^2$.

Everything else:

- n calls to makeSet
- $2m$ calls to find: 2 calls per edge to find its endpoints.
- $n-1$ calls to union: A tree has $n-1$ edges.

Total: $O((m+n)\log(n))$. For connected graphs = $O(m\log(n))$.

Fast-Kruskal(G = (V,E)):
  for $v \in V$, makeSet(v)
  for edges (u,v) ∈ E in increasing order of weight
    If find$(v) \neq$ find(u)
      $X \leftarrow X \cup \{(u,v)\}$
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Kruskal's Algorithm with Connected Components

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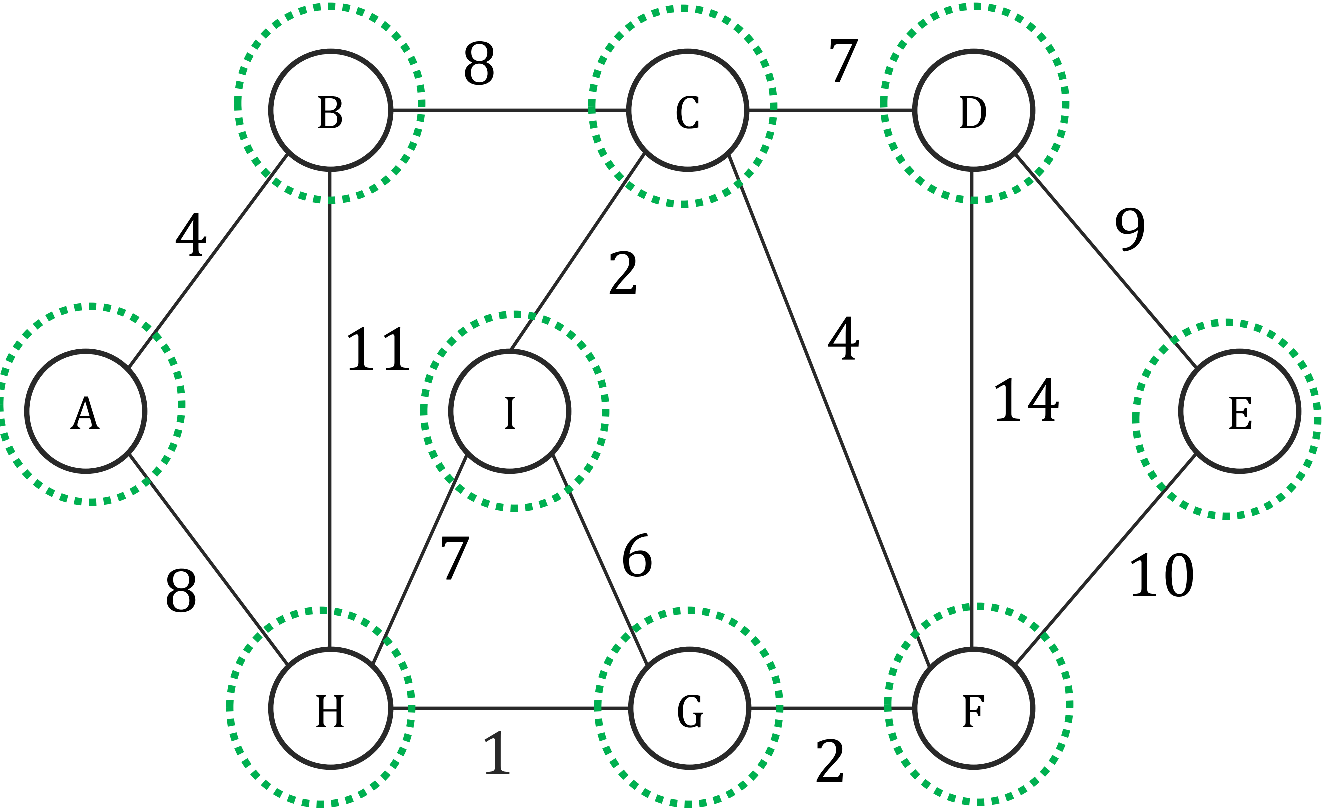
Below, we highlight the connected components. Each refer to one set in Union-Find Data structure.



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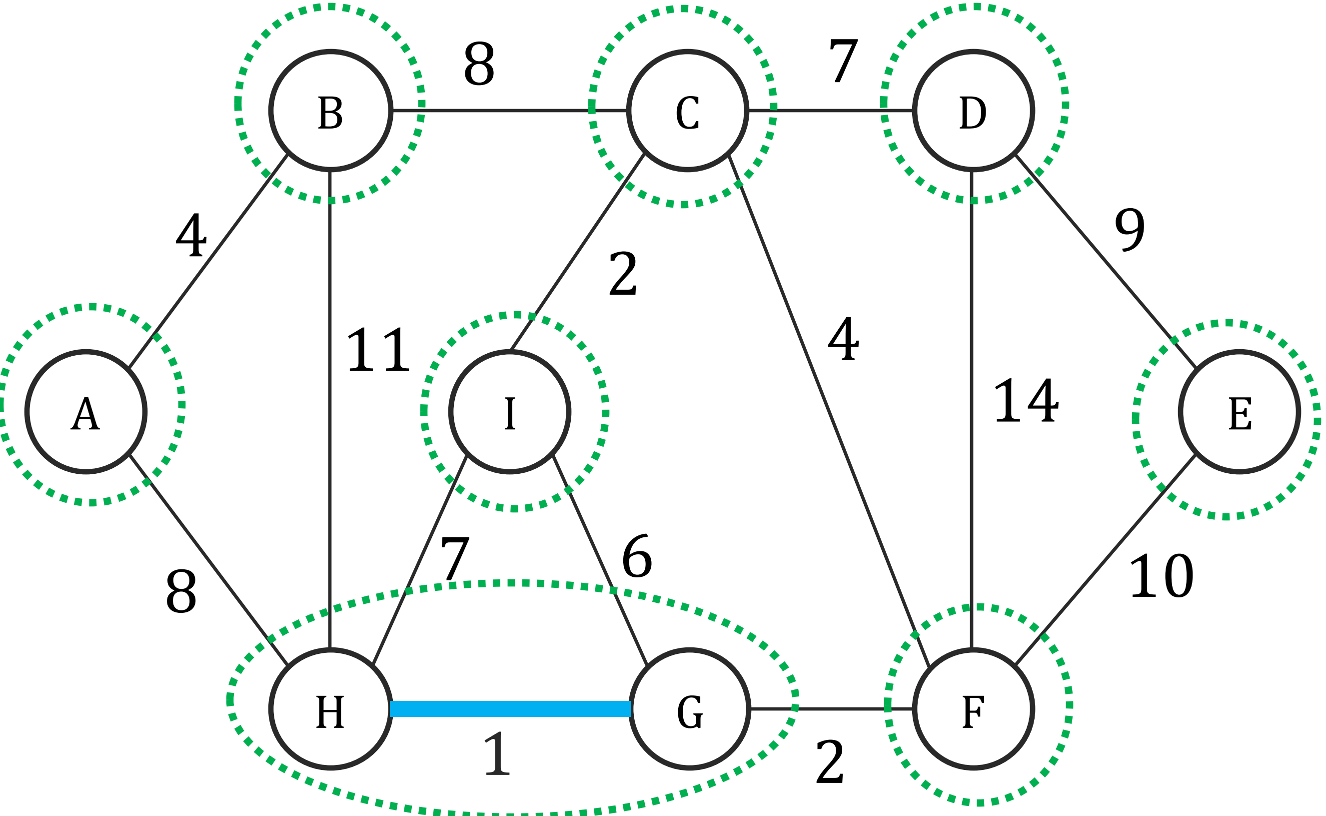

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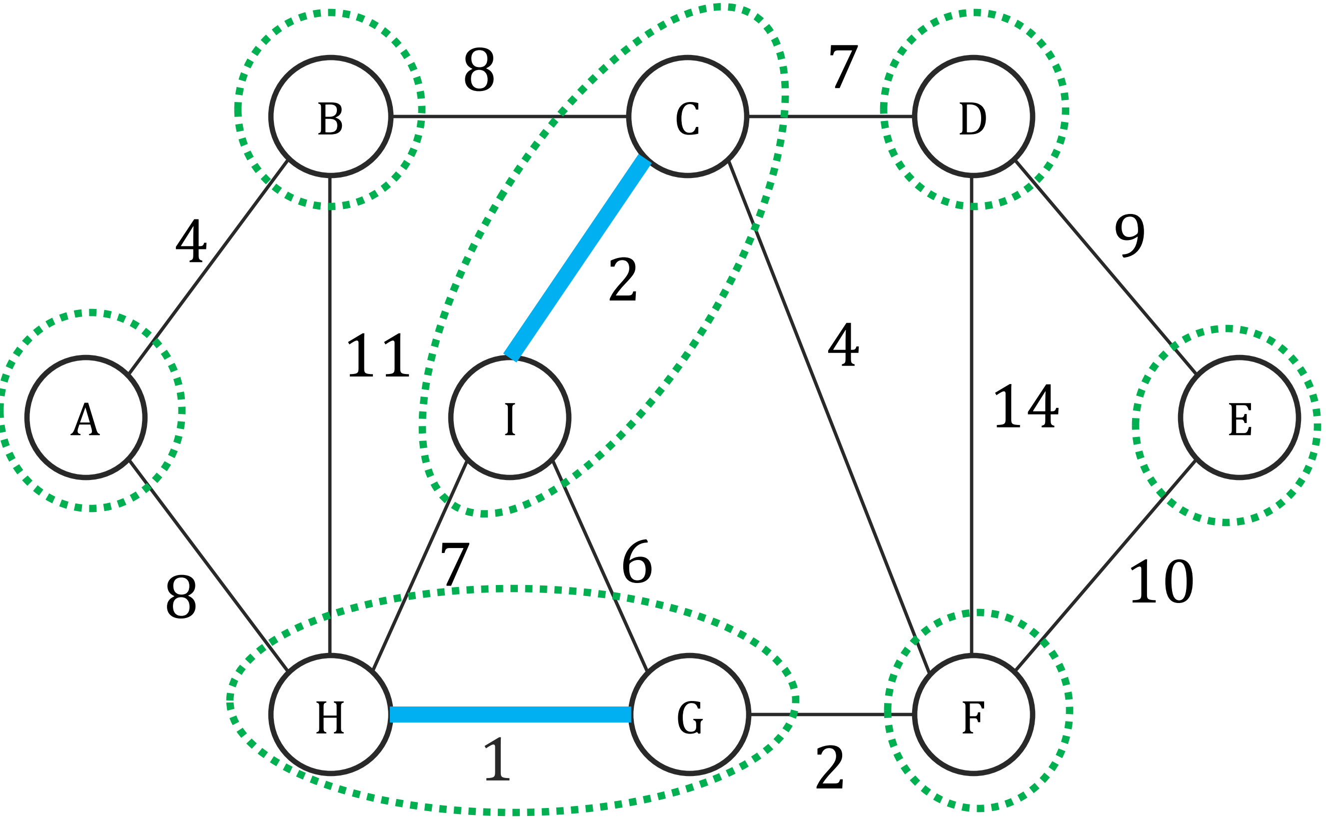

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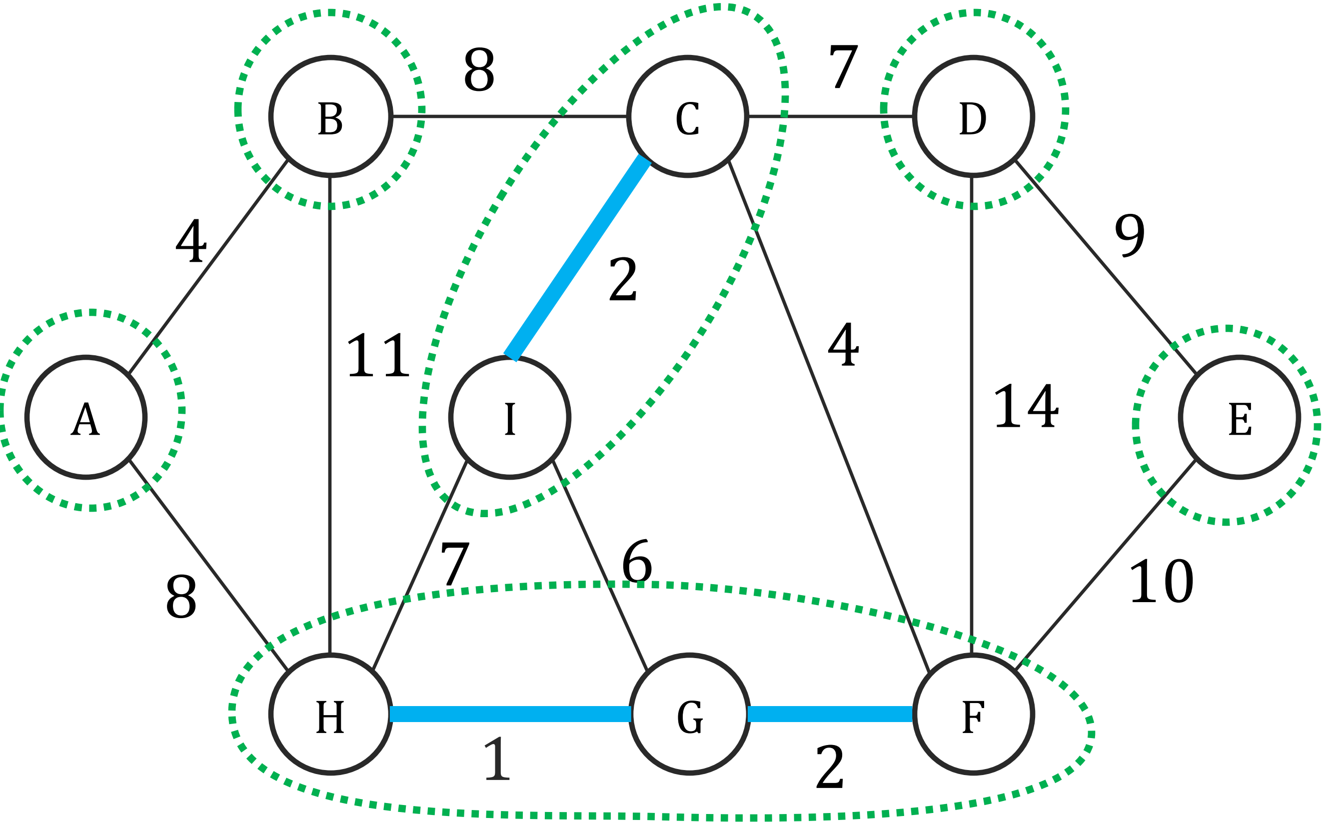

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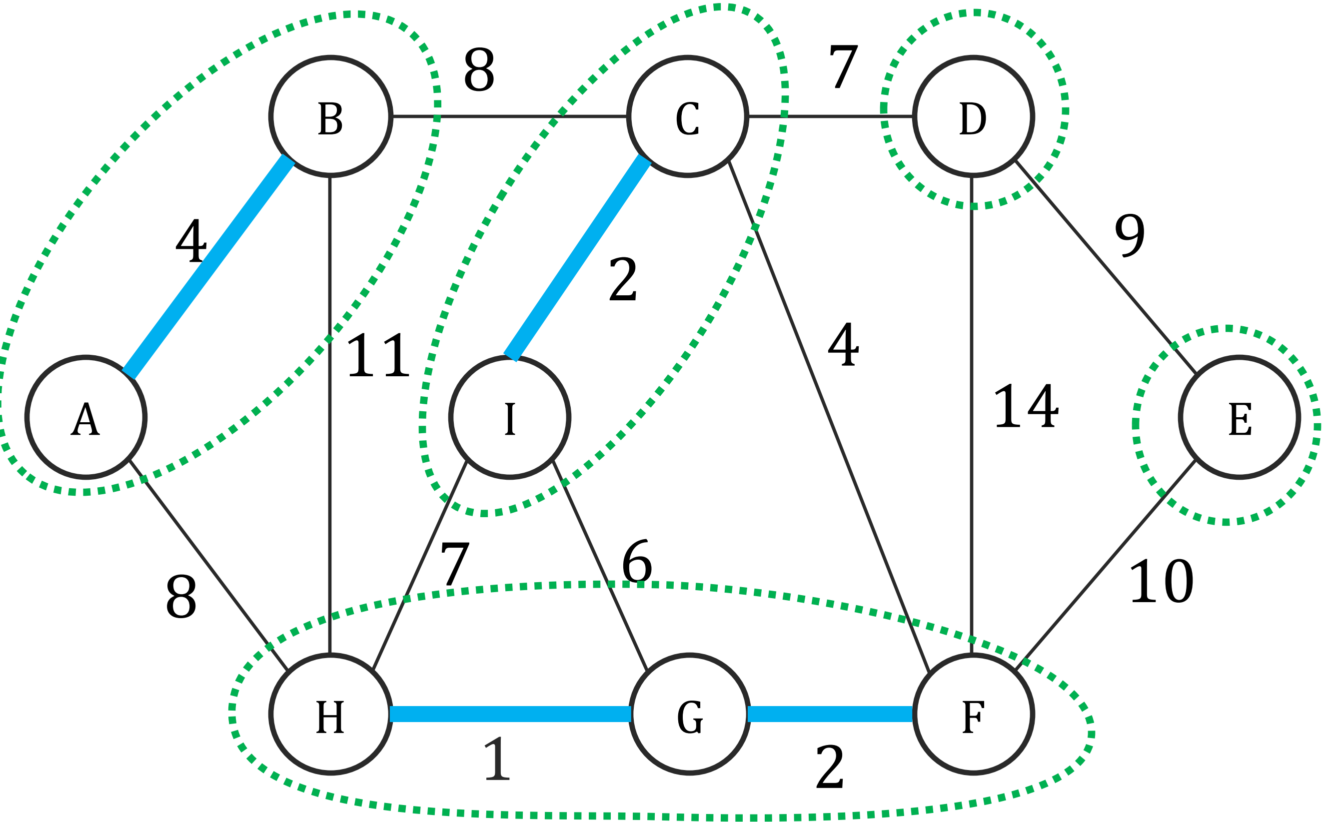

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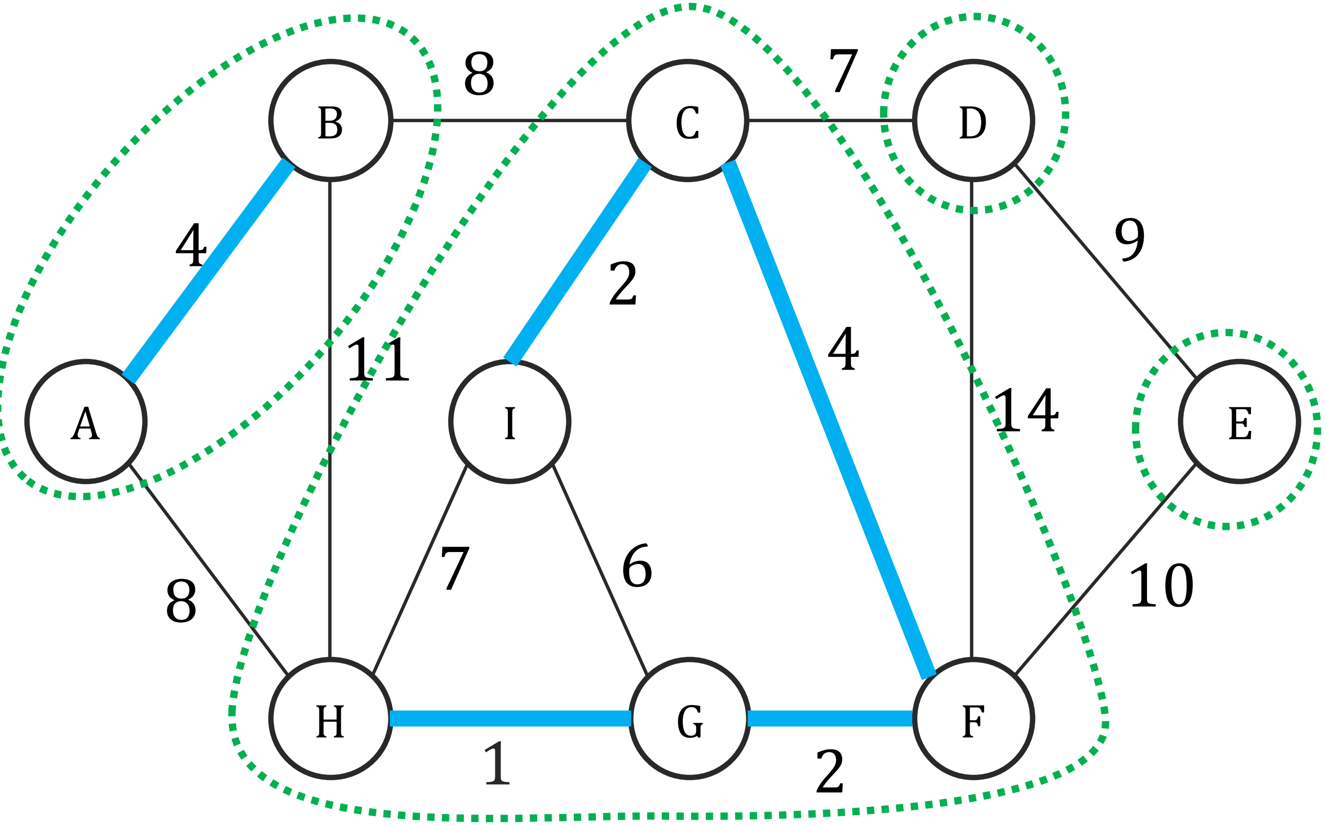

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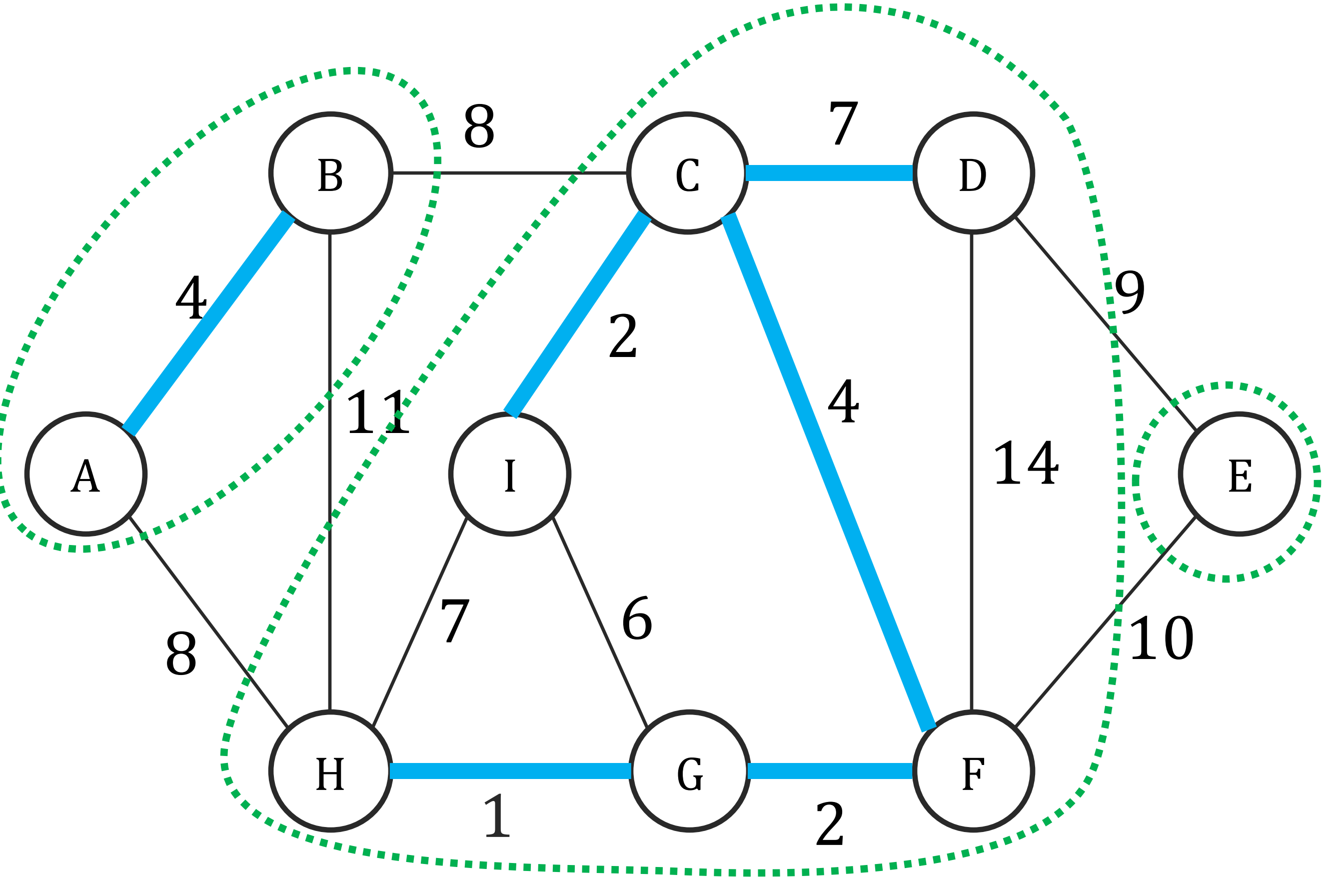

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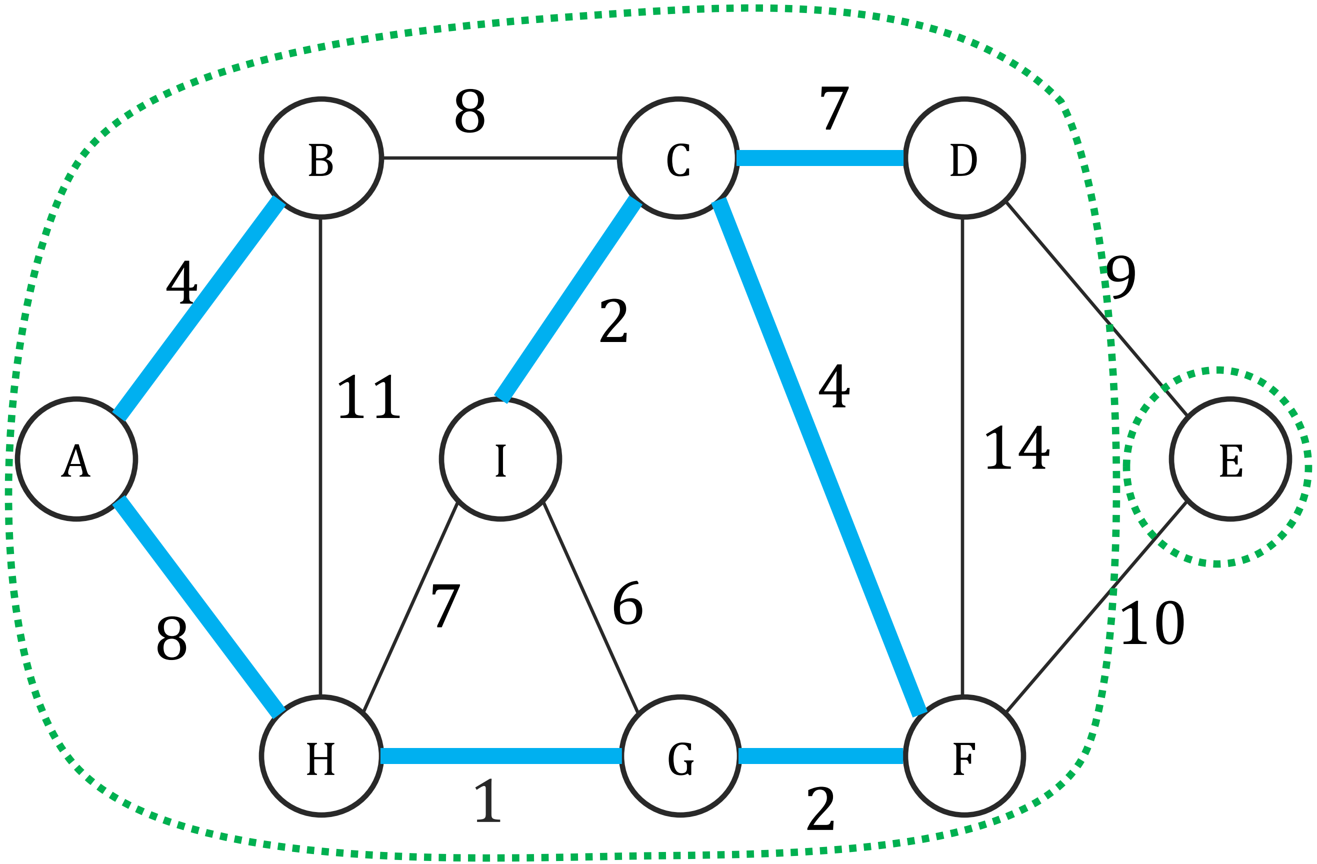

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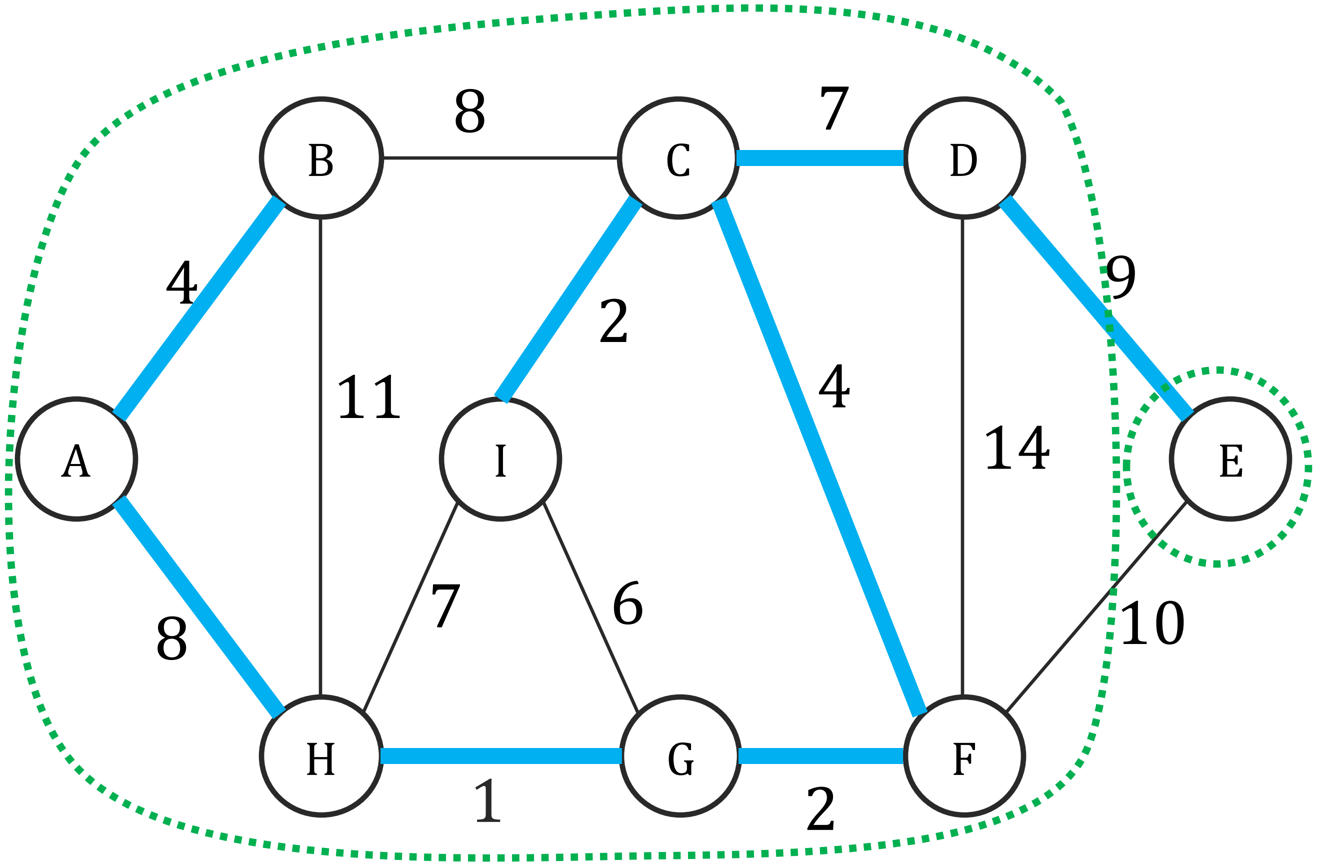

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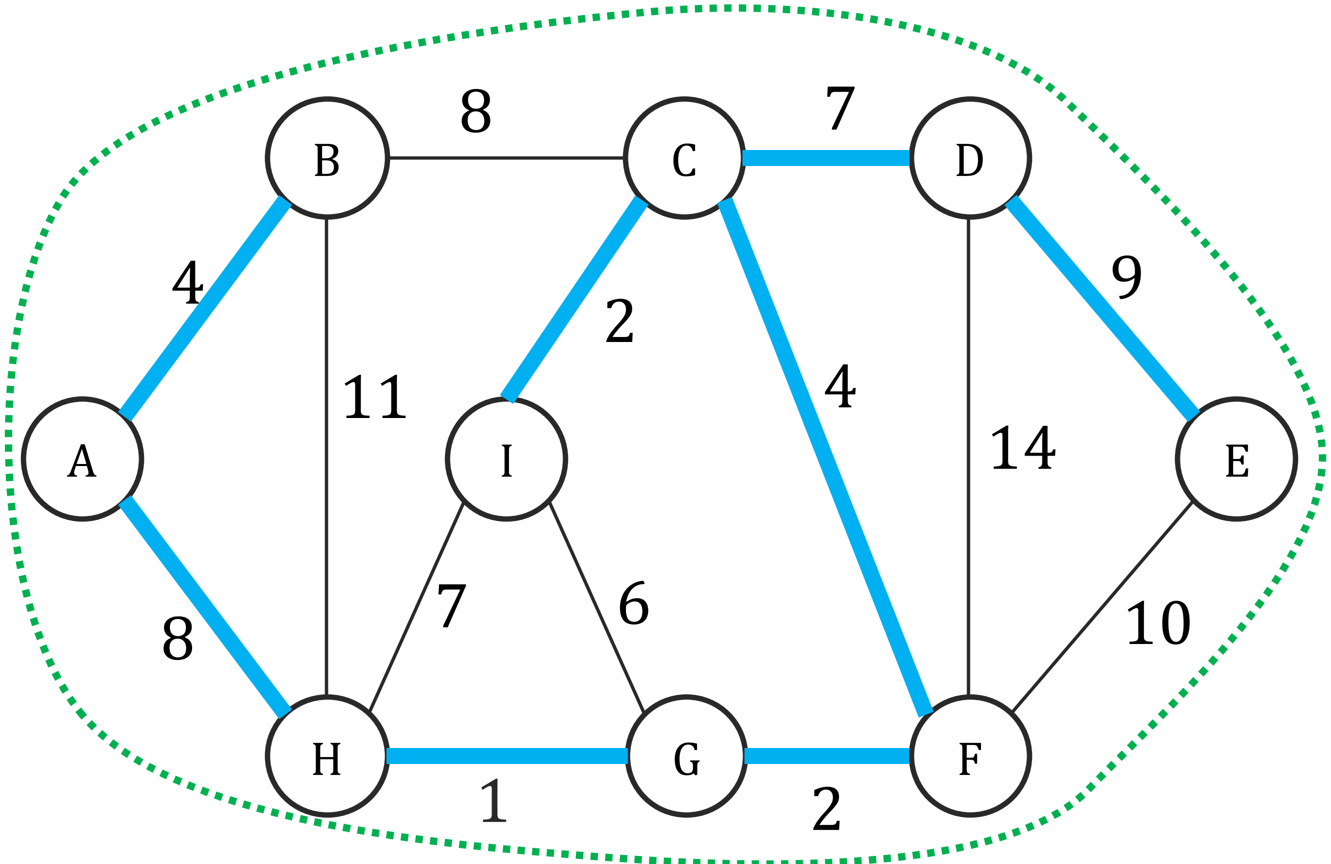

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Prim's Algorithm
A different MST greedy algorithm

A different greedy algorithm for MSTs

Idea:

- Keep X connected at all times, so S is the connected component representing X.
- Grow a tree greedily by adding the cheapest edge that can grow the tree.

"Cut Property":
If X is a subset of an MST and has no edges from S to $V \setminus S$, then $X \cup \{e\}$ is also a subset of an MST.

Meta Algorithm for MST

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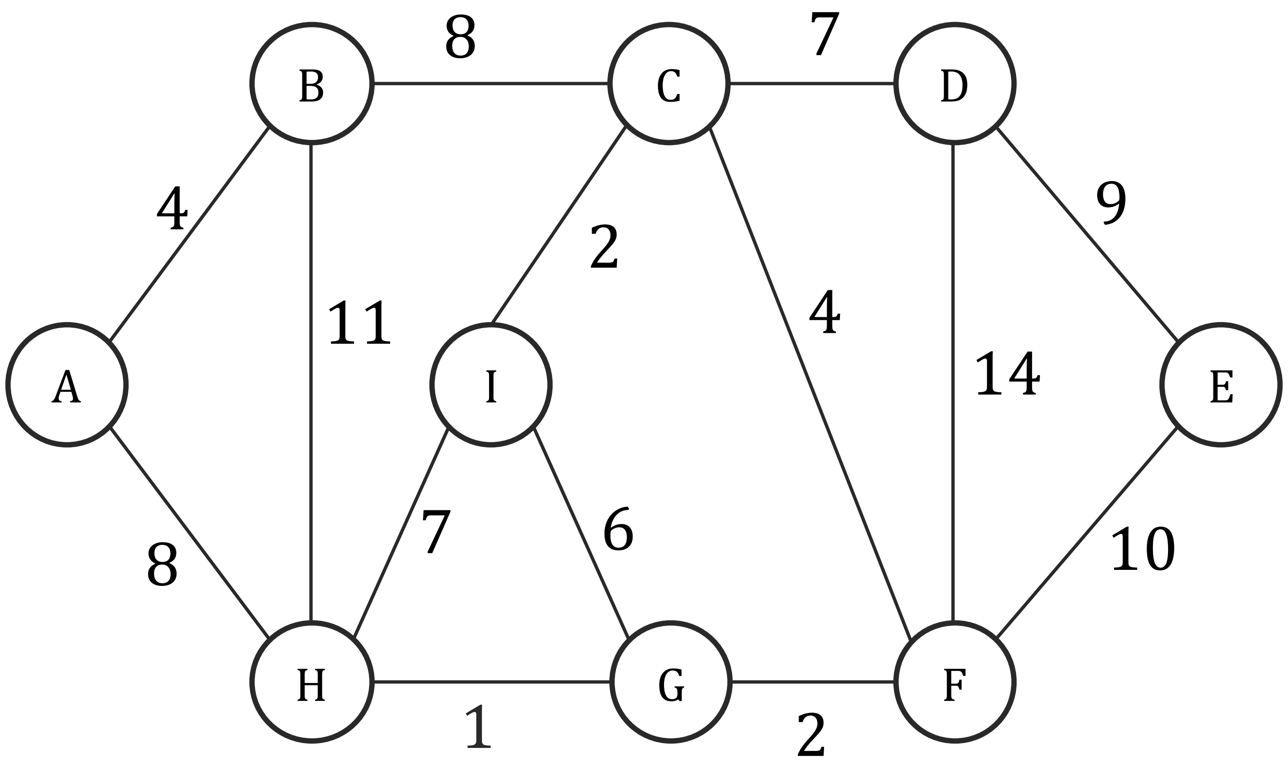
Grow a tree greedily by adding the cheapest edge that can grow the tree.



Prim$(G = (V, E))$

$S \leftarrow \{A\}$ // an arbitrary node A.

$X = \{\}$

while $|X| < |V| - 1$

Let $e = (u, v)$ be the lightest edge such that $u \in S$ and $v \in V \setminus S$.

$X \leftarrow X \cup \{e\}$

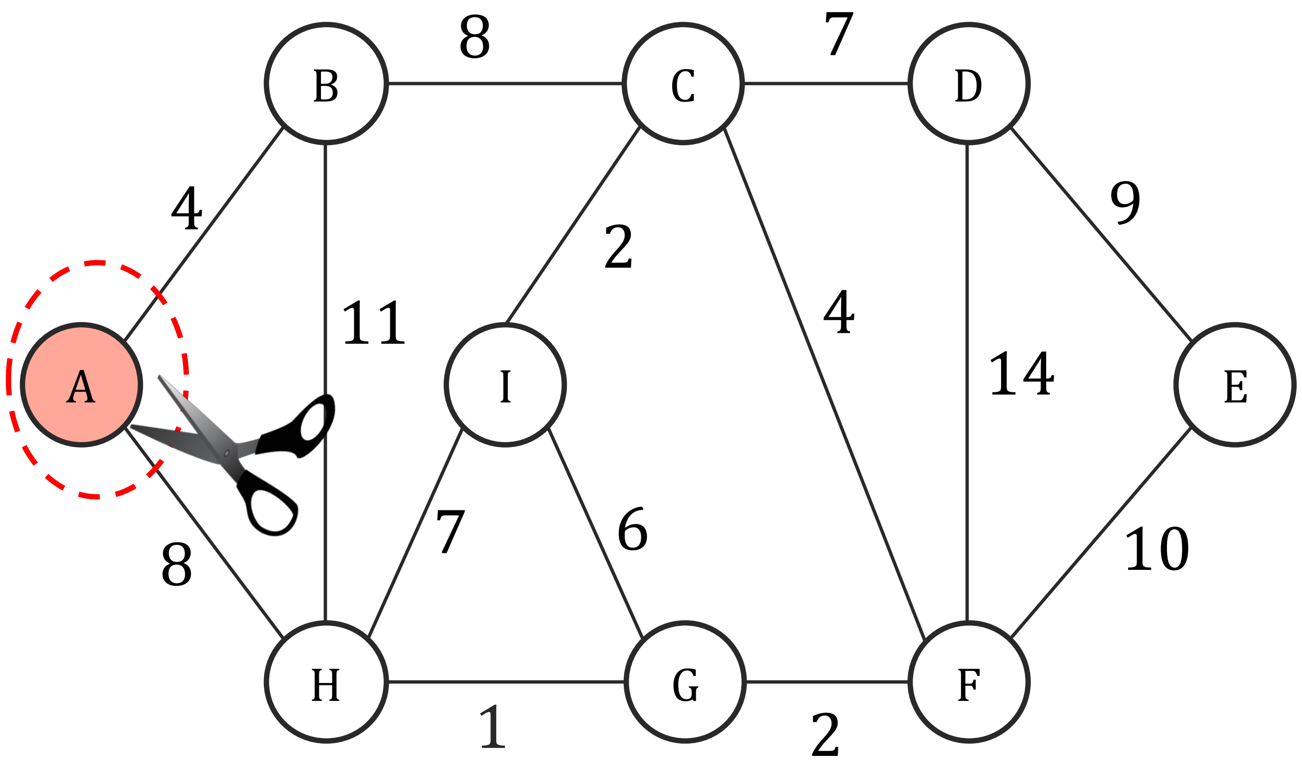
$S \leftarrow S \cup \{v\}$

Return X

Prim's Algorithm

Grow a tree greedily by adding the cheapest edge that can grow the tree.

Red dotted line indicates the set S.



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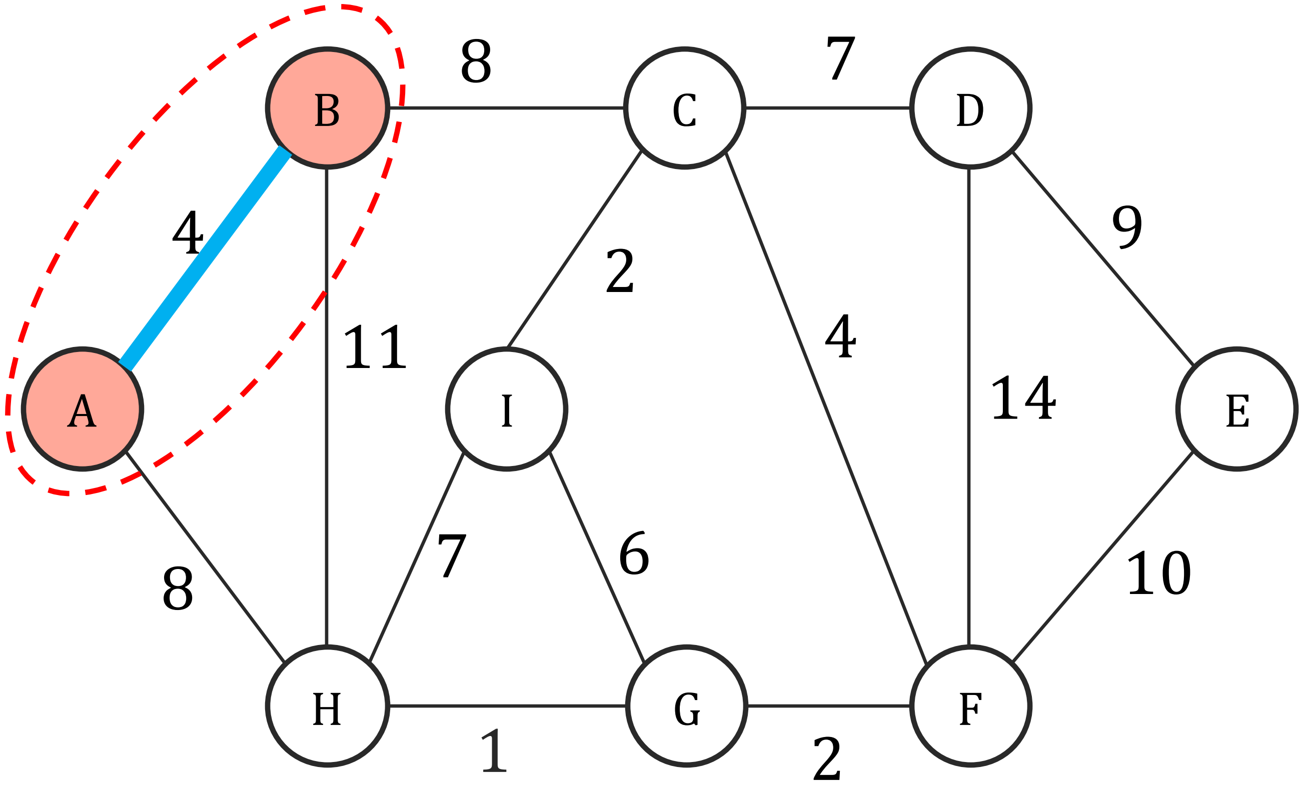

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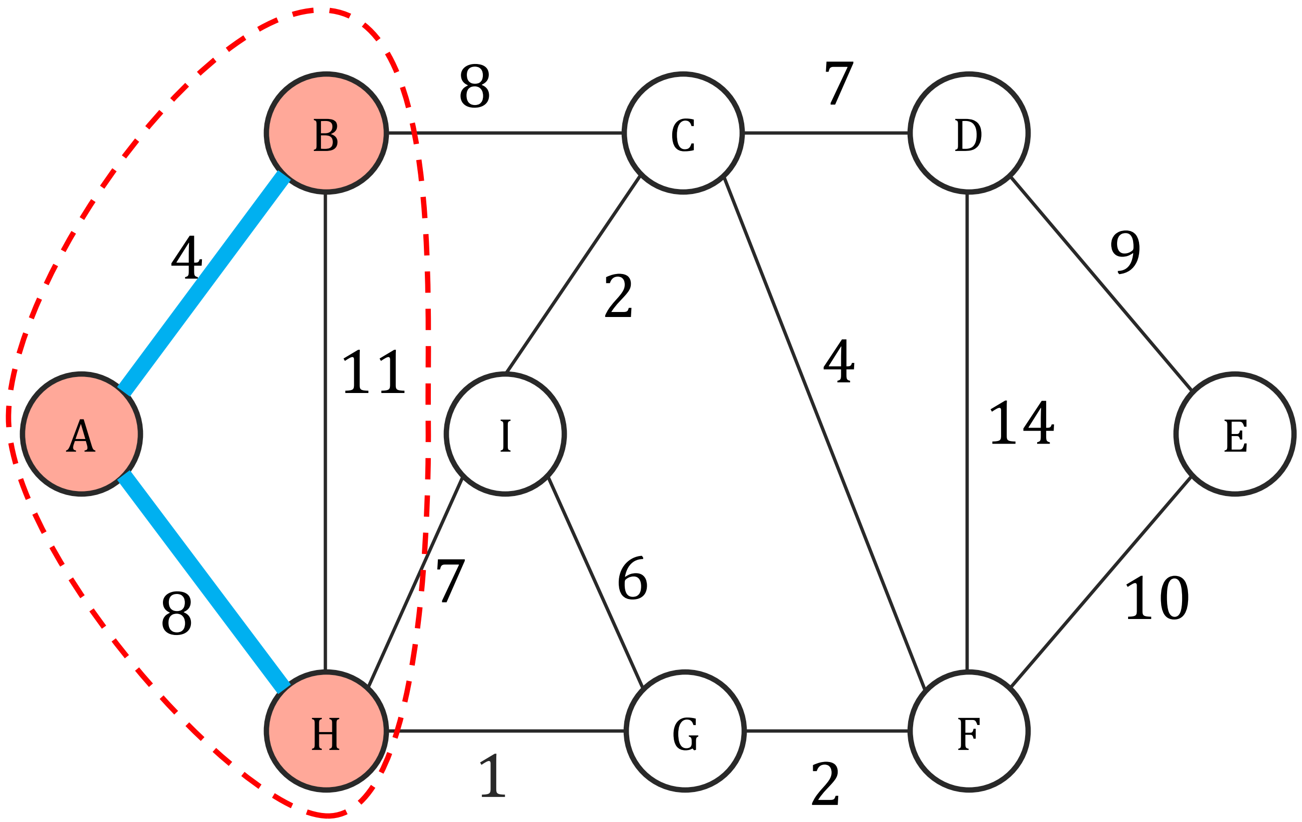

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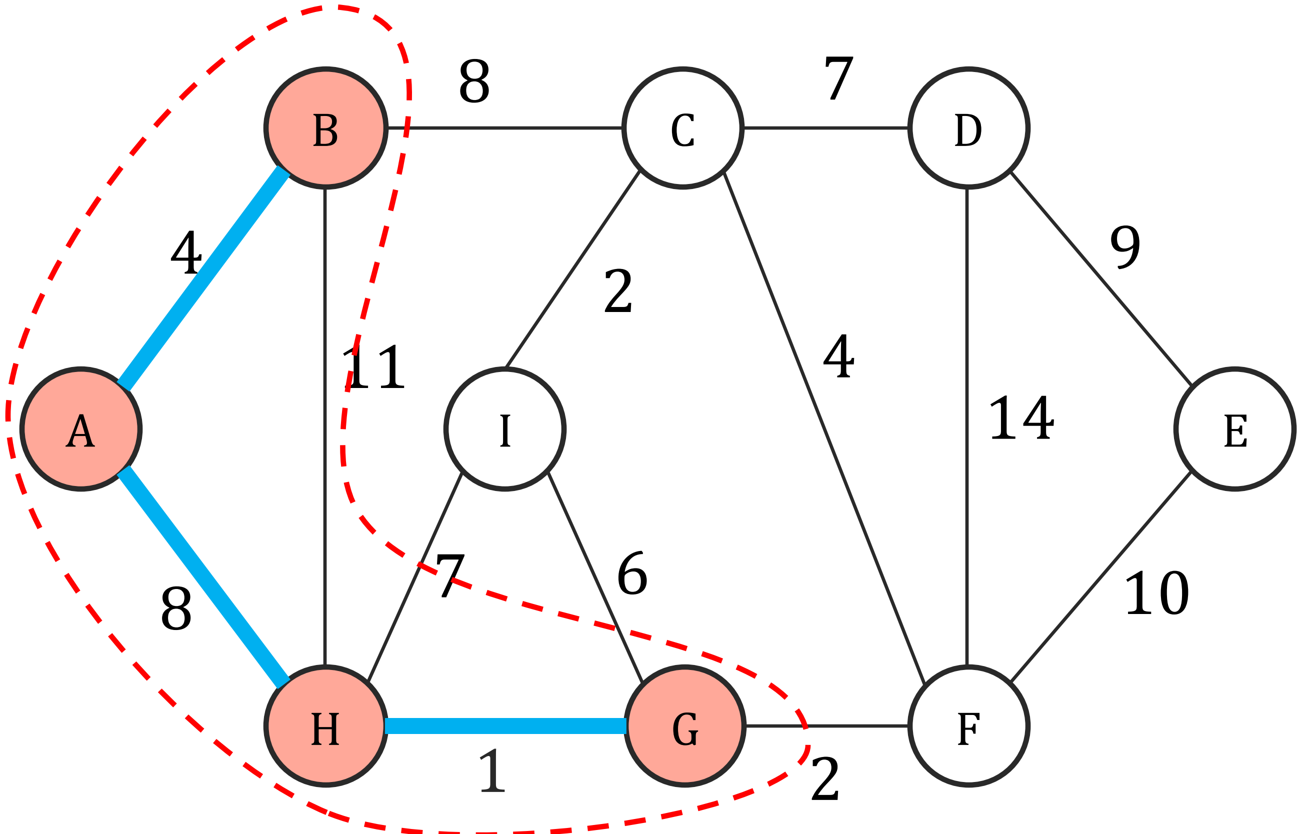

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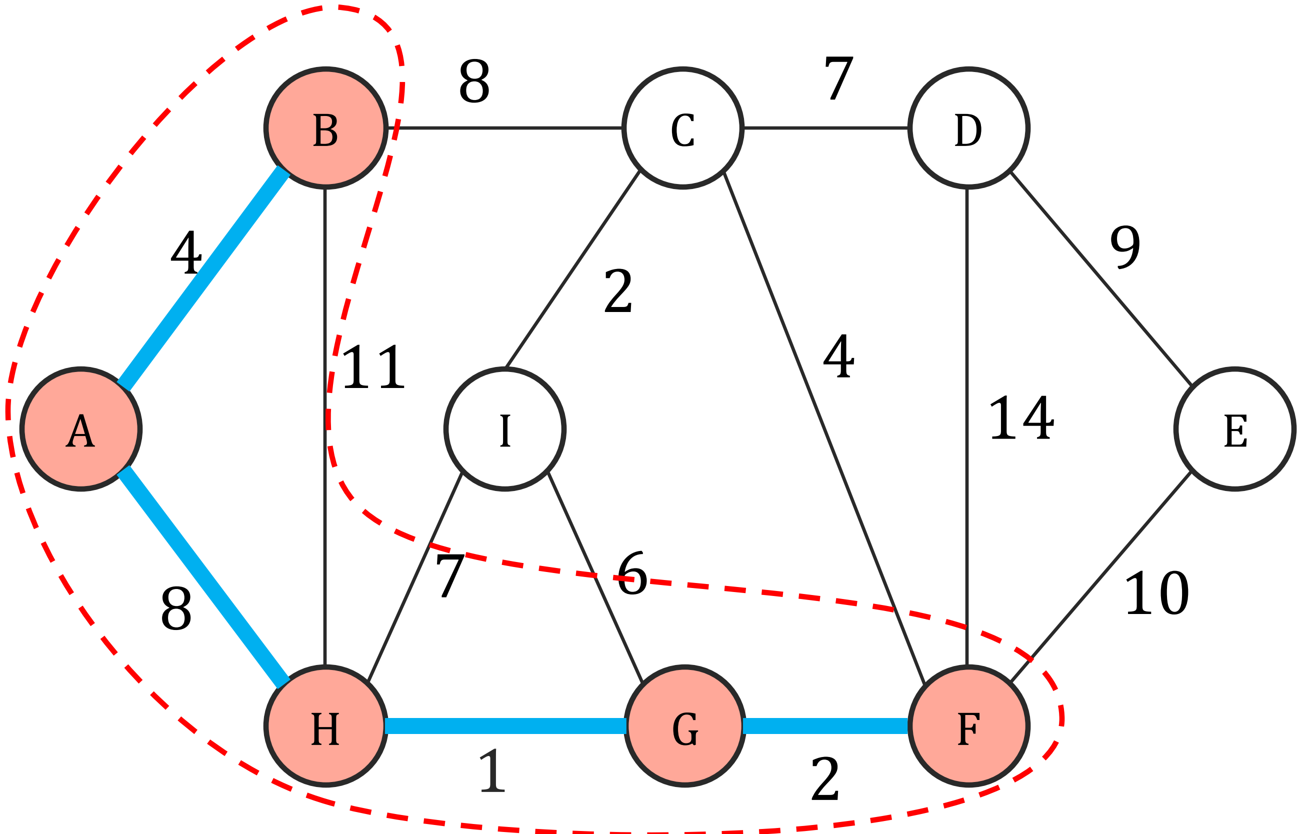

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Prim's Algorithm

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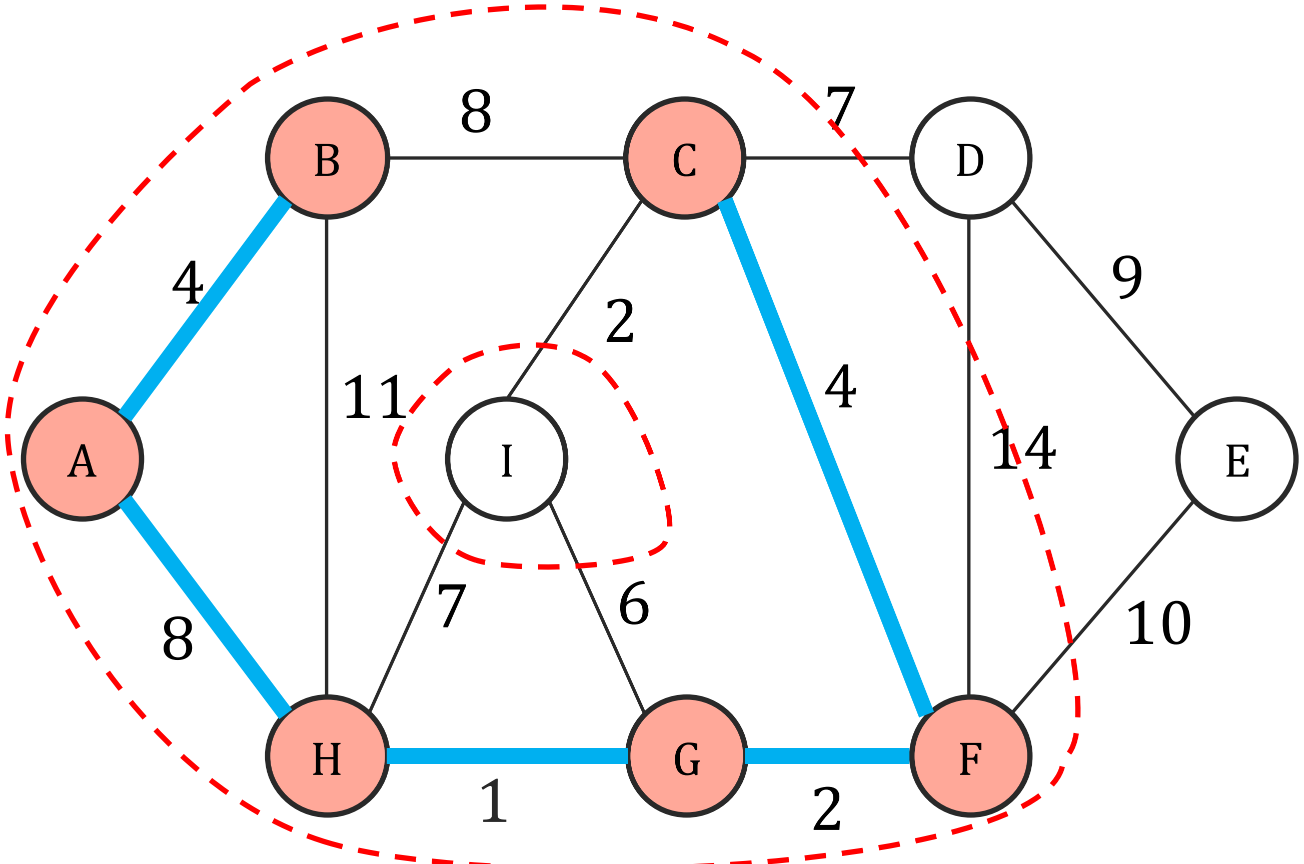
Here, we choose a donut to visually represent the set S so only edges crossing from S to $V \setminus S$ visually cross the dotted line.



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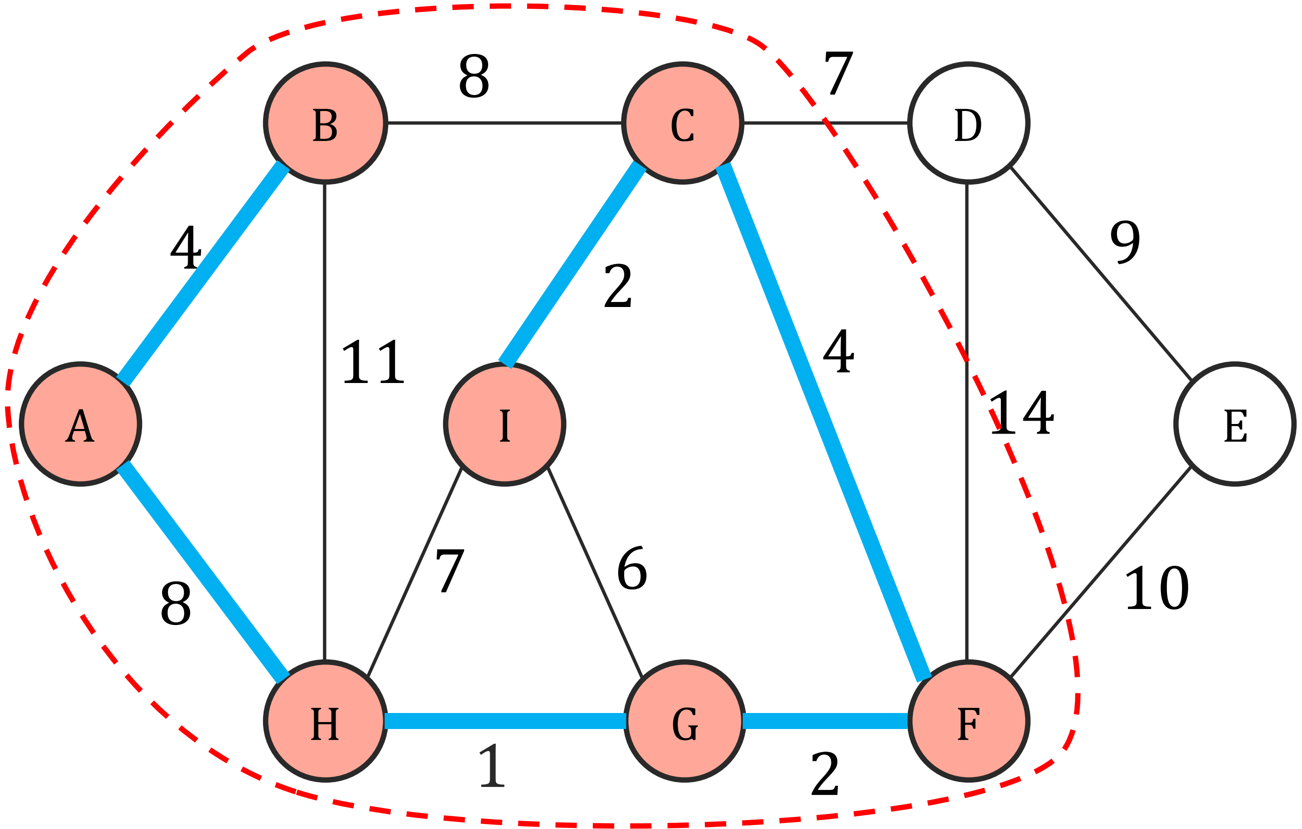
Red dotted line indicates the set S.



Prim$(G = (V, E))$

$S \leftarrow \{A\}$ // an arbitrary node A.

$X = \{\}$

while $|X| < |V| - 1$

Let $e = (u, v)$ be the lightest edge such that $u \in S$ and $v \in V \setminus S$.

$X \leftarrow X \cup \{e\}$

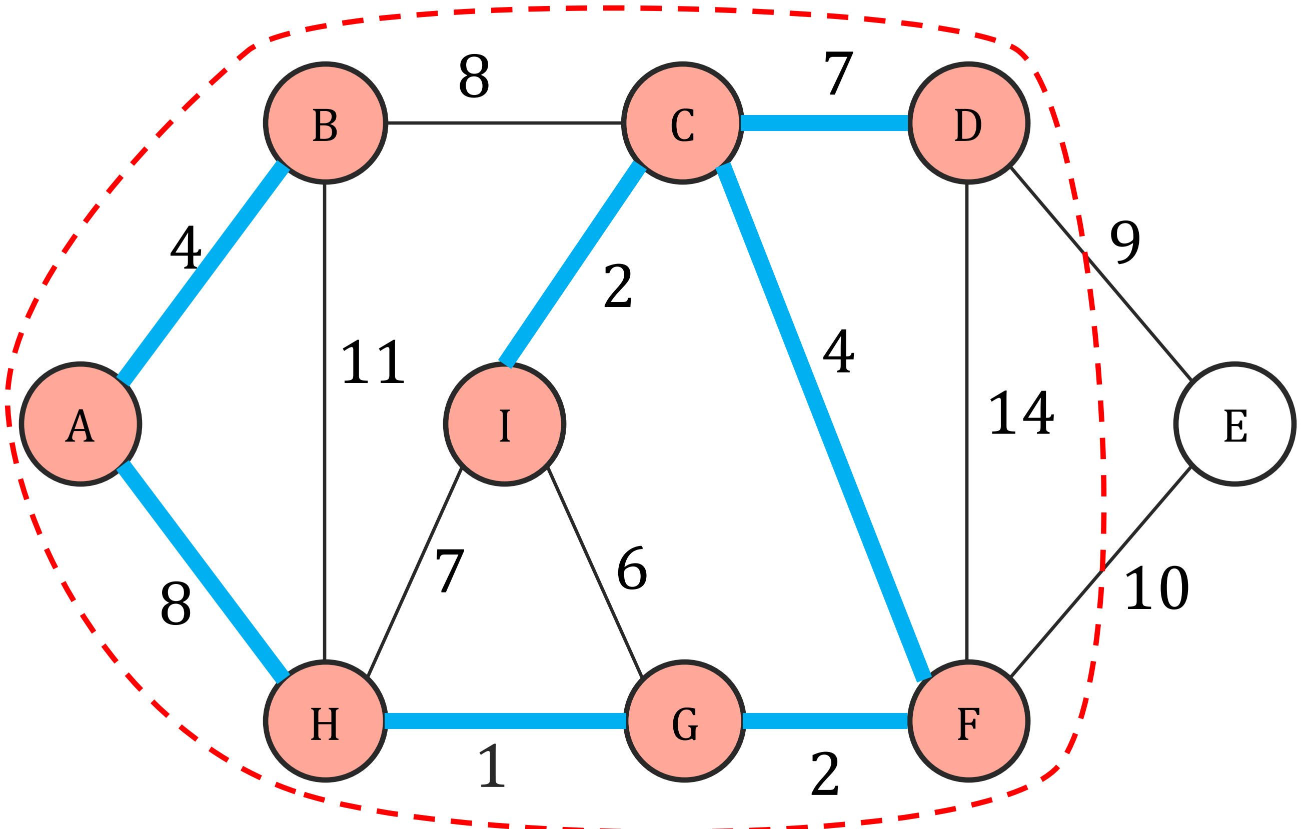
$S \leftarrow S \cup \{v\}$

Return X

Prim's Algorithm

Grow a tree greedily by adding the cheapest edge that can grow the tree.

Red dotted line indicates the set S.



Prim$(G = (V, E))$

$S \leftarrow \{A\}$ // an arbitrary node A.

$X = \{\}$

while $|X| < |V| - 1$

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Prim's Algorithm

Grow a tree greedily by adding the cheapest edge that can grow the tree.

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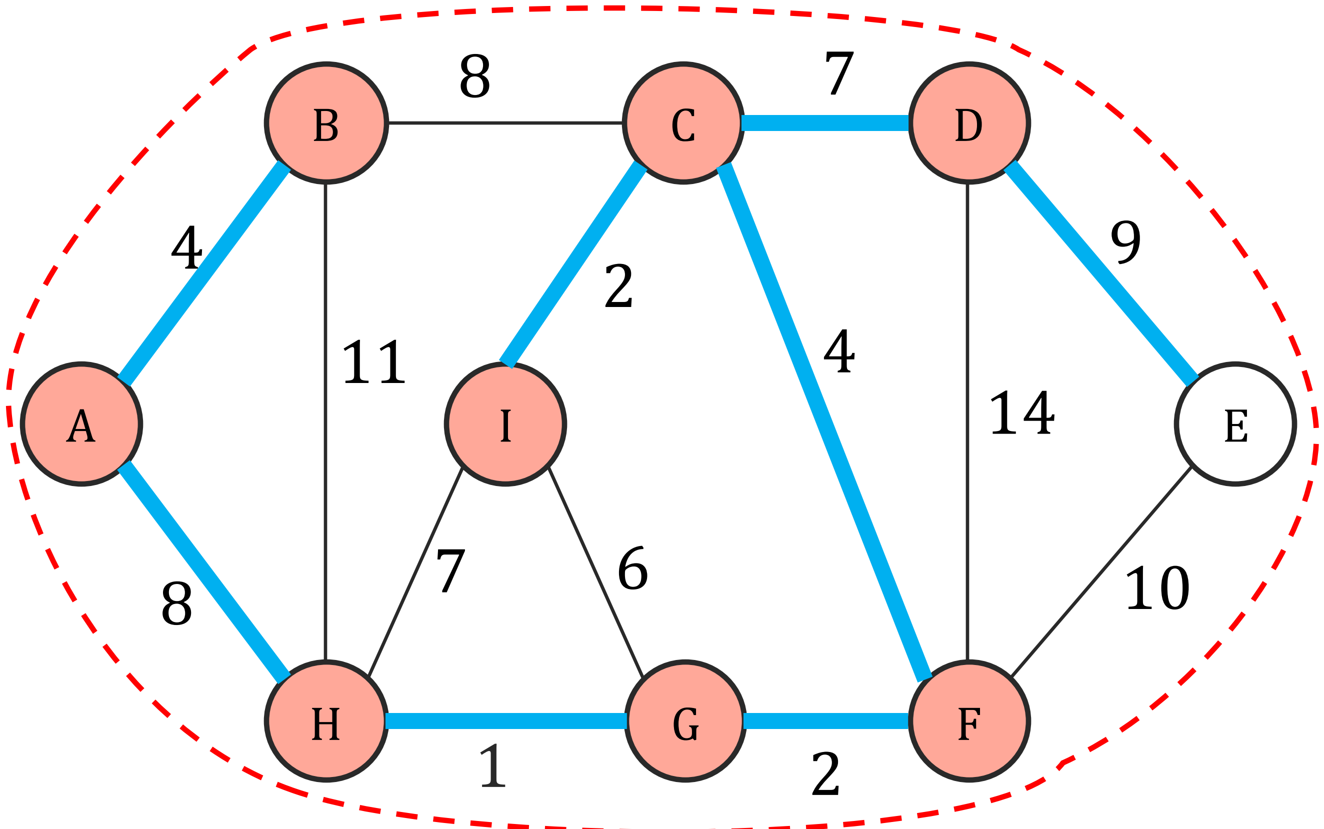


Prim$(G = (V, E))$

  $S \leftarrow \{A\}$ // an arbitrary node A.

  $X = \{\}$

  while $|X| < |V| - 1$

    Let $e = (u, v)$ be the lightest edge

      such that $u \in S$ and $v \in V \setminus S$.

    $X \leftarrow X \cup \{e\}$

    $S \leftarrow S \cup \{v\}$

Return X

Correctness of Prim's Algorithm

Does Prim's Algorithm return a minimum spanning tree?

- X forms a tree and S refers to the set of vertices connected by this tree.
- Only edges that can "grow" a tree are those that go from S to $V \setminus S$

→At every step, Prim adds the lightest such edge.

So, Prim's algorithm fits the meta algorithm description, so it find an MST.

Meta Algorithm for MST

$X = \{\}$
Repeat until $|X| = |V| - 1$
    Pick $S \subset V$, s.t. X has no edges from S to $V \setminus S$
    $e \leftarrow$ lightest weight edge from S to $V \setminus S$
    $X \leftarrow X \cup \{e\}$

How to implement Prim's Algorithm

This pseudo-code seems very slow!

At most $n-1$ iterations of this while loop.

Runtime of at most m to go through all the edges and find the lightest.

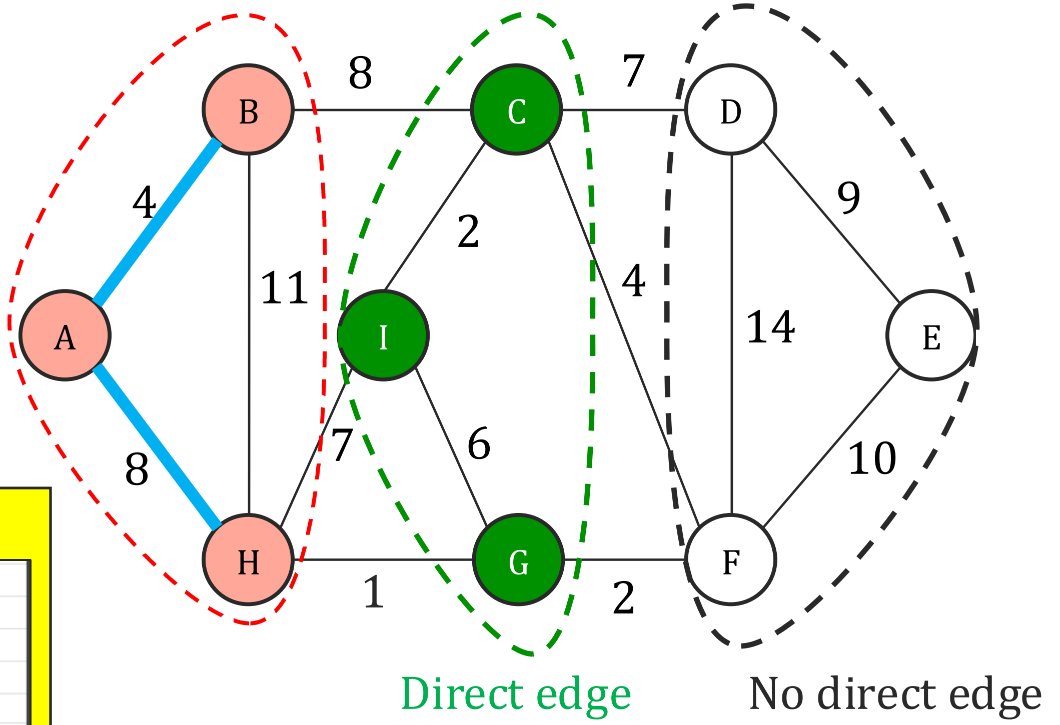
Naively implementing this, take $O(nm)$.

Prim($G = (V, E)$)
  $S \leftarrow \{A\}$ // an arbitrary node A.
  X = {}
  while $|X| < |V| - 1$
    Let $e = (u, v)$ be the lightest edge
      such that $u \in S$ and $v \in V \setminus S$.
    $X \leftarrow X \cup \{e\}$
    $S \leftarrow S \cup \{v\}$
Return X

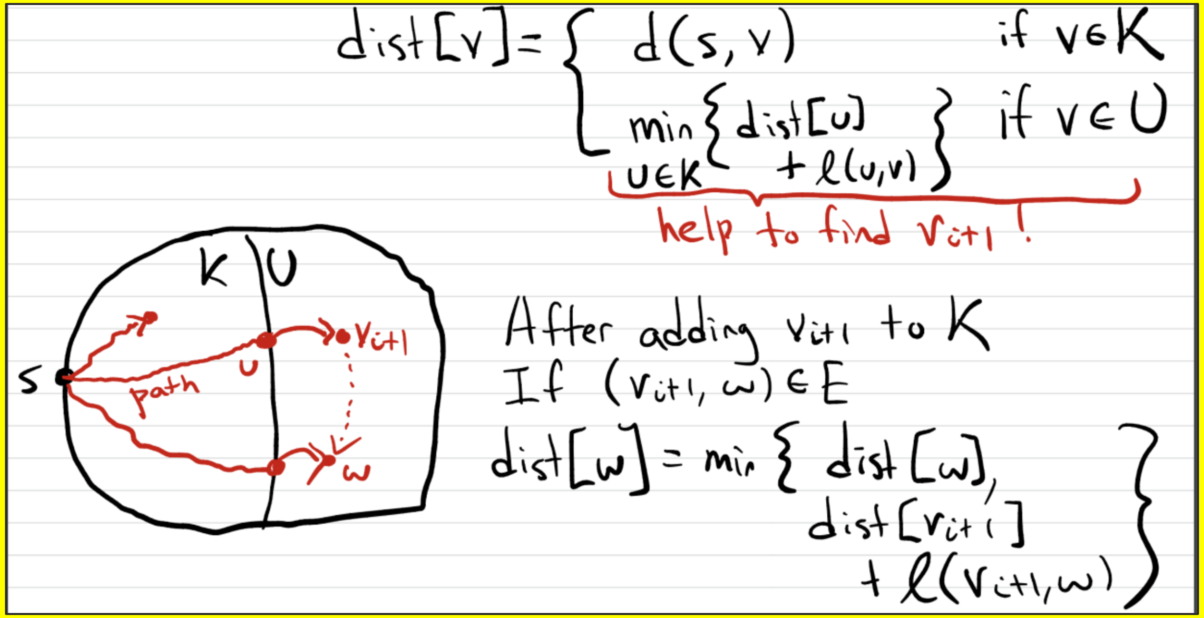
How do we actually implement Prim's Algorithm?

For each vertex $v \in V \setminus S$, we need to keep track of

- Whether v has direct edge the set S of "visited" vertices.
- The cost of the lightest edge connecting v to the set S of "visited" vertices.



Same dilemma in lecture 7!



Implementing Prim's Algorithm Fast

We use the same idea as we did for Dijkstra's, with small changes.

Each vertex has

- cost $dist[v]$ instantiated to ∞ and pointer $prev[v]$ instantiated to null
  - →If a neighbor u is added to the visited set S and $dist[v] > w_{(u,v)}$:

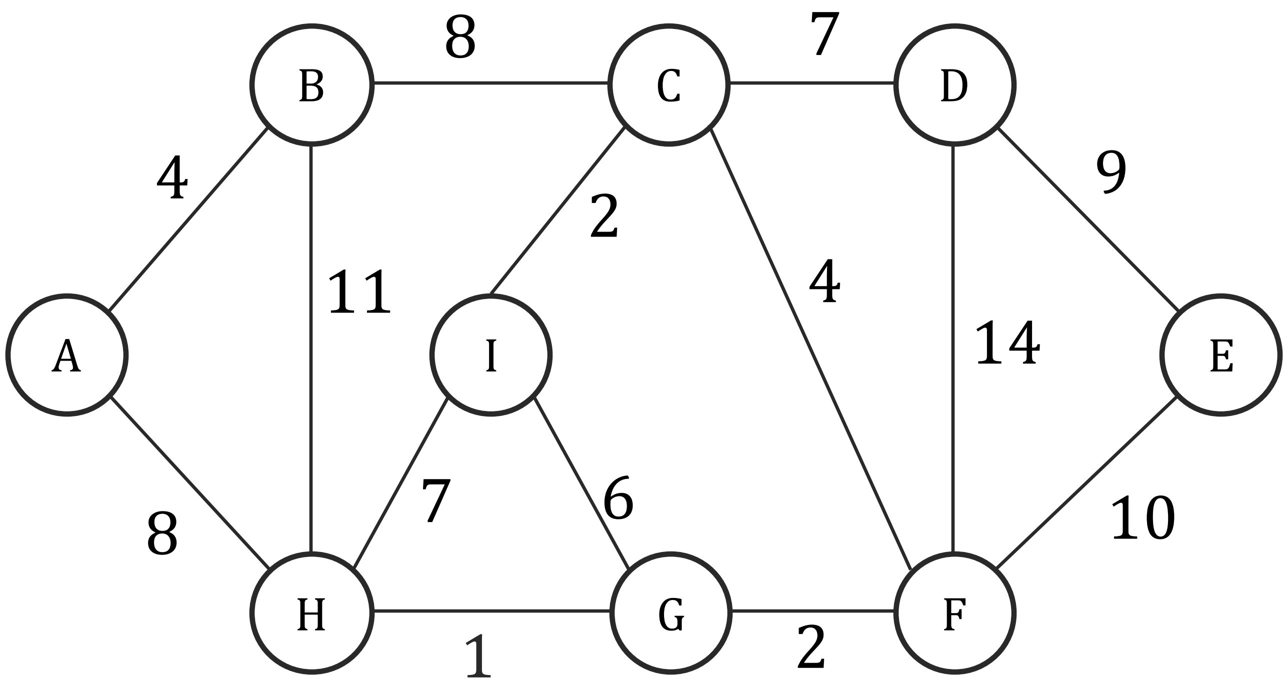
update $dist[v] \leftarrow w_{(u,v)}$.

update $prev[v] \leftarrow u$

How is this different from Dijkstra?

- In Dijkstra, the condition to perform an update and the update accounted for the entire length of s-v path
  - → e.g., if $dist[v] > dist[u]+w_{(u,v)}$, then update $dist[v] \leftarrow dist[u]+w_{(u,v)}$
- Here, we only care about distance to the closest visited node, not the entire path.

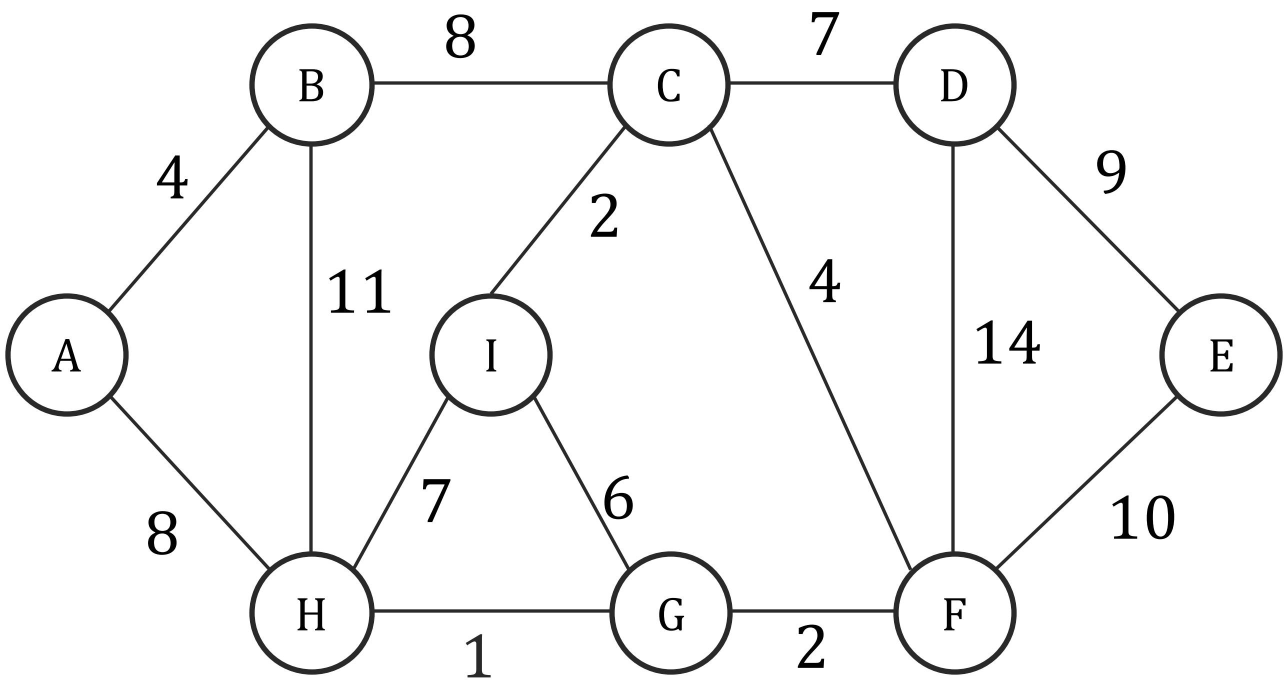
Prim's Algorithm: Efficient Implementation



	A	B	C	D	E	F	G	H	I
dist									
prev									

Fast-Prim$(G = (V, E))$
    array $dist(n)$ // initialize to all ∞
    array $prev(n)$ // initialized to null
    $X = \{\}$ and Q empty priority queue
    $dist[A] = 0$ // an arbitrary node A
    for $v \in V$, Q. insert$(v, dist[v])$
    while $|X| < |V| - 1$
        $v \leftarrow$ Q.deleteMin
        if $v \neq A$, $X \leftarrow X \cup \{(prev[v], v)\}$
        for $(v, z) \in E$
            if $dist[z] > w_{(v,z)}$ and $z \in Q$.
                Q. decreaseKey$(z, w_{(v,z)})$
                $prev[z] \leftarrow v$
    return X

Prim's Algorithm: Efficient Implementation



	A	B	C	D	E	F	G	H	I
dist	0	∞	∞	∞	∞	∞	∞	∞	∞
prev	∅	∅	∅	∅	∅	∅	∅	∅	∅

Fast-Prim$(G = (V, E))$

  array $dist(n)$ // initialize to all ∞

  array $prev(n)$ // initialized to null

  $X = \{\}$ and Q empty priority queue

  $dist[A] = 0$ // an arbitrary node A

  for $v \in V$, Q. insert$(v, dist[v])$

  while $|X| < |V| - 1$

    $v \leftarrow$ Q.deleteMin

    if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

    for $(v, z) \in E$

      if $dist[z] > w_{(v,z)}$ and $z \in Q$.

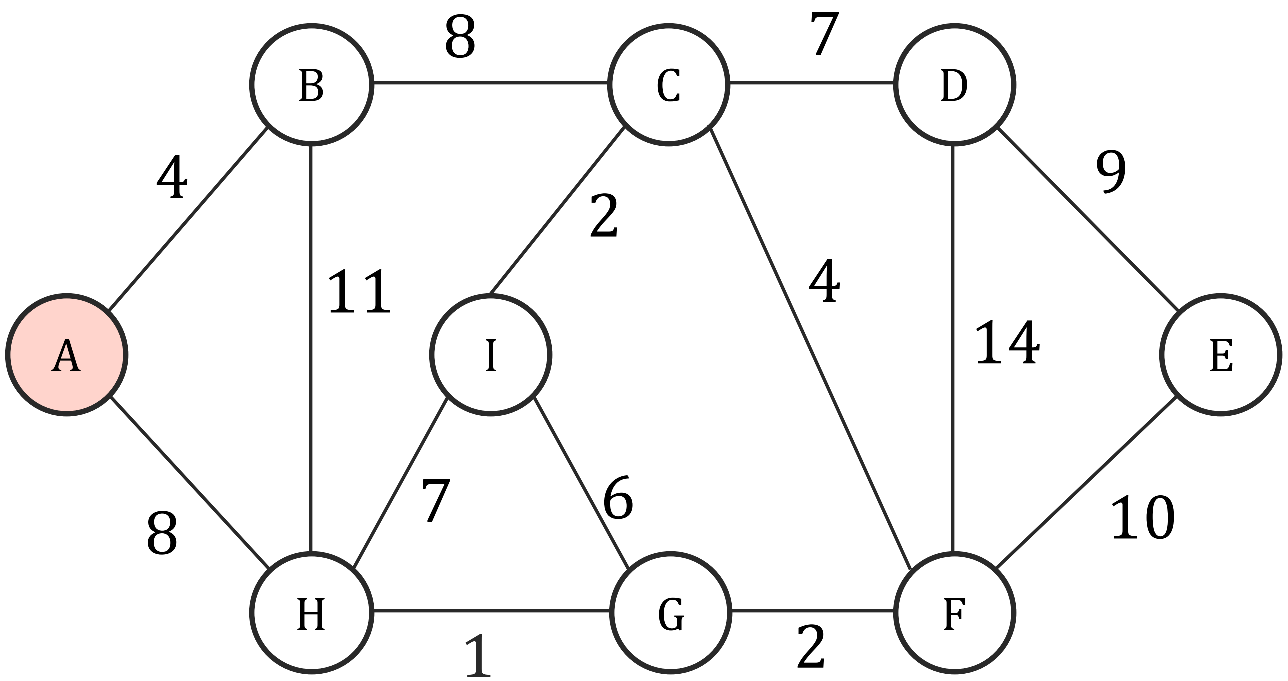
        Q. decreaseKey$(z, w_{(v,z)})$

        $prev[z] \leftarrow v$

  return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q

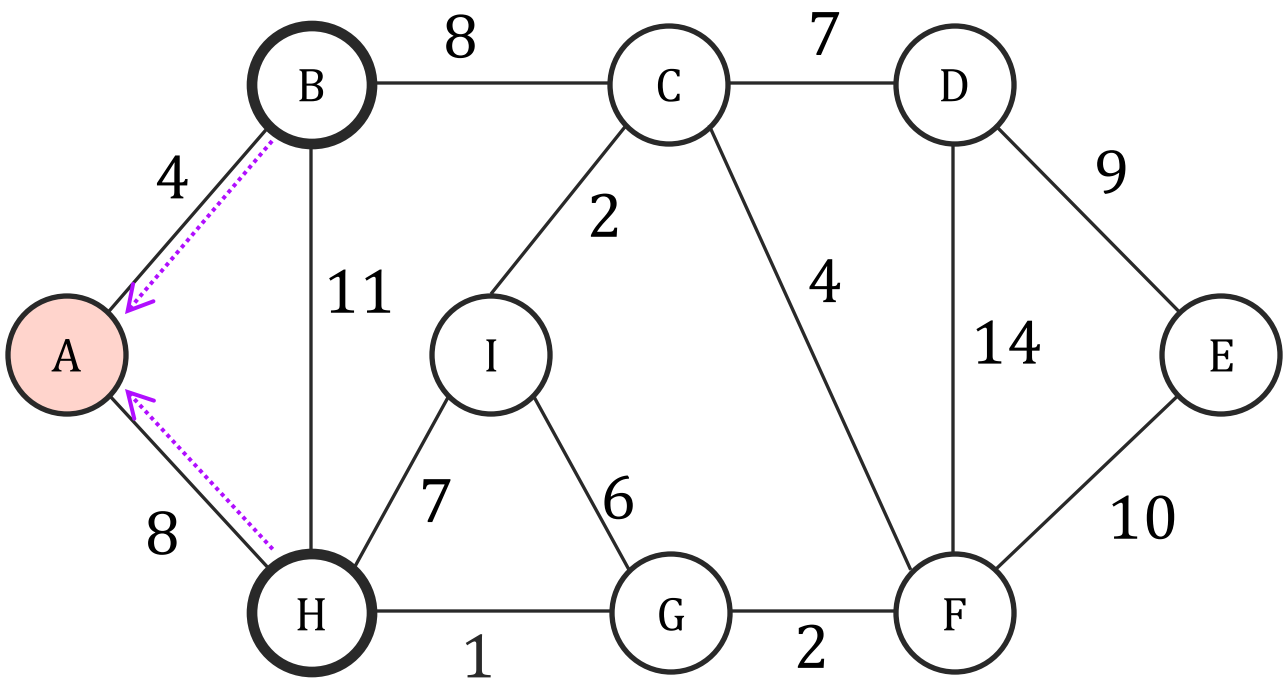


	A	B	C	D	E	F	G	H	I
$dist$	0	∞	∞	∞	∞	∞	∞	∞	∞
$prev$	∅	∅	∅	∅	∅	∅	∅	∅	∅

Fast-Prim$(G = (V, E))$
- array $dist(n)$ // initialize to all ∞
- array $prev(n)$ // initialized to null
- $X = \{\}$ and Q empty priority queue
- $dist[A] = 0$ // an arbitrary node A
- **for** $v \in V$, Q.insert$(v, dist[v])$
- **while** $|X| < |V| - 1$
  - $v \leftarrow$ Q.deleteMin A
  - **if** $v \neq A$, $X \leftarrow X \cup \{(prev[v], v)\}$
  - **for** $(v, z) \in E$
    - **if** $dist[z] > w_{(v,z)}$ and $z \in Q$.
      - Q.decreaseKey$(z, w_{(v,z)})$
      - $prev[z] \leftarrow v$
- **return** X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	**B**	C	D	E	F	G	**H**	I
dist	0	**4**	∞	∞	∞	∞	∞	**8**	∞
prev	∅	***A***	∅	∅	∅	∅	∅	***A***	∅

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

array $prev(n)$ // initialized to null

$X = \{\}$ and Q empty priority queue

$dist[A] = 0$ // an arbitrary node A

for $v \in V$, $Q.\text{insert}(v, dist[v])$

while $|X| < |V| - 1$

$v \leftarrow$ Q.deleteMin

if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

for $(v, z) \in E$

if $dist[z] > w_{(v,z)}$ and $z \in Q$.

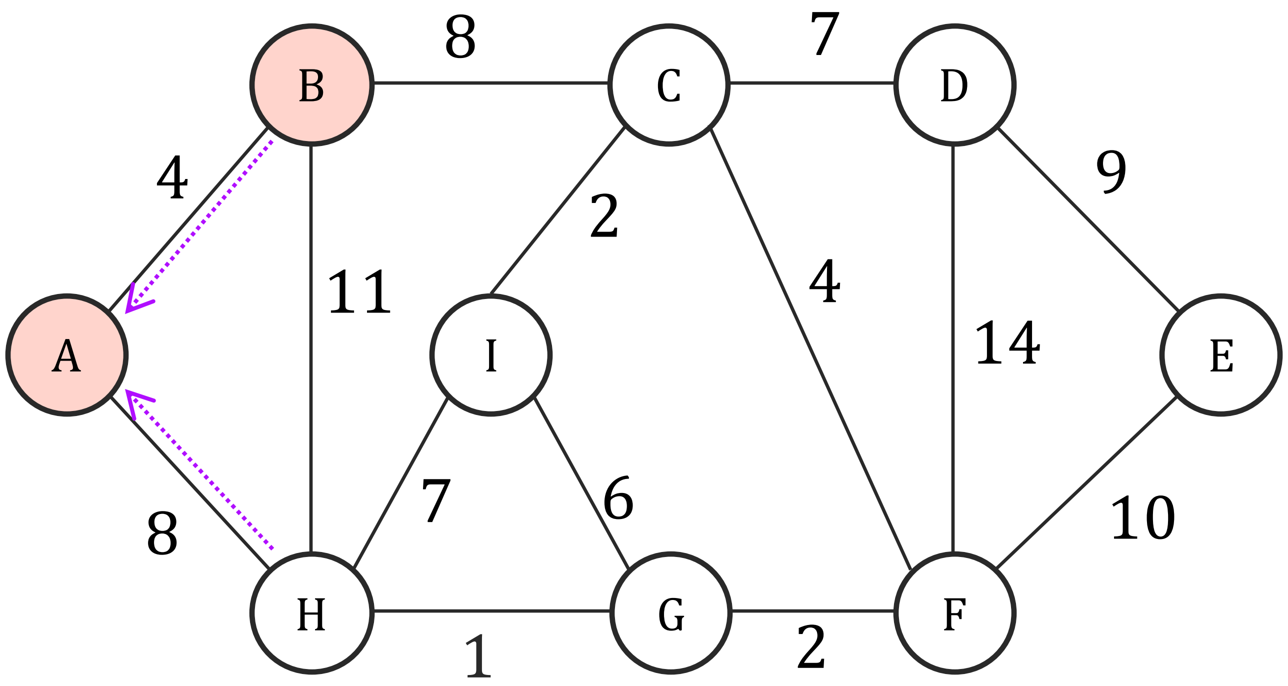
$Q.\text{decreaseKey}(z, w_{(v,z)})$

$prev[z] \leftarrow v$

return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	∞	∞	∞	∞	∞	8	∞
prev	∅	*A*	∅	∅	∅	∅	∅	*A*	∅

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

array $prev(n)$ // initialized to null

$X = \{\}$ and Q empty priority queue

$dist[A] = 0$ // an arbitrary node A

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while $|X| < |V| - 1$

$v \leftarrow$ Q.deleteMin

if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

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if $dist[z] > w_{(v,z)}$ and $z \in Q$.

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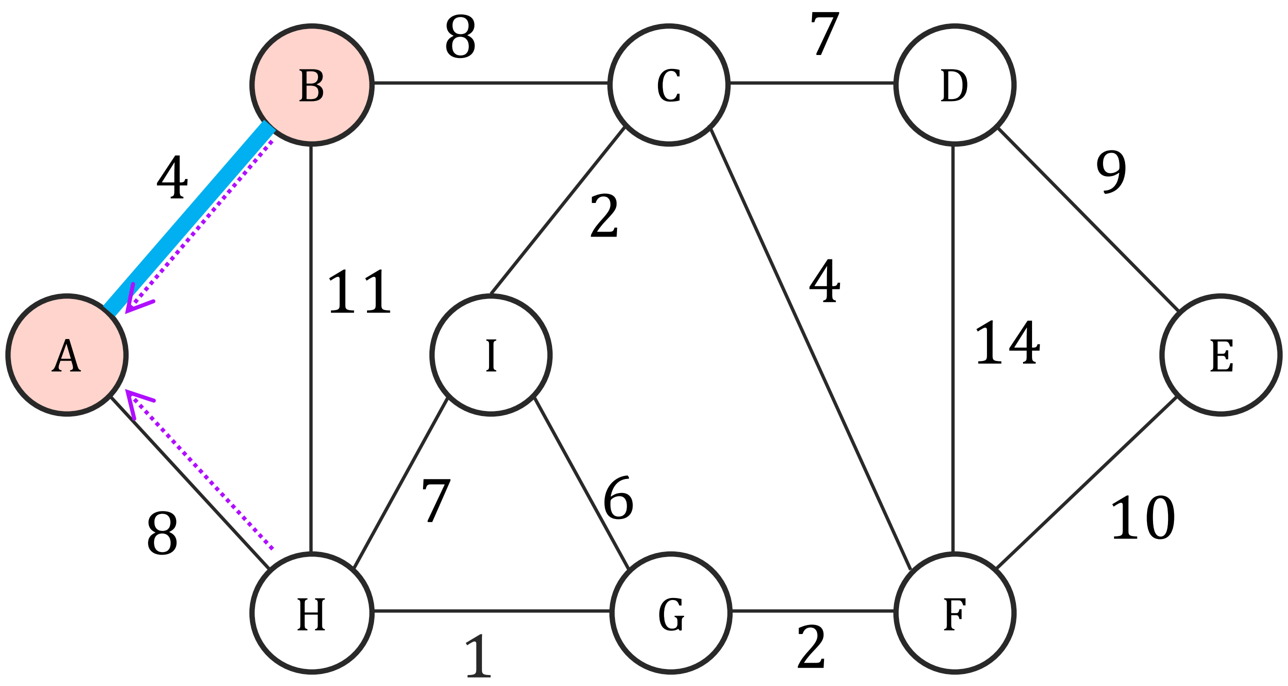
$prev[z] \leftarrow v$

return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	∞	∞	∞	∞	∞	8	∞
prev	∅	*A*	∅	∅	∅	∅	∅	*A*	∅

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    $v \leftarrow$ Q.deleteMin
    if $v \neq A$, $X \leftarrow X \cup \{(prev[v], v)\}$
    for $(v, z) \in E$
      if $dist[z] > w_{(v,z)}$ and $z \in Q$.
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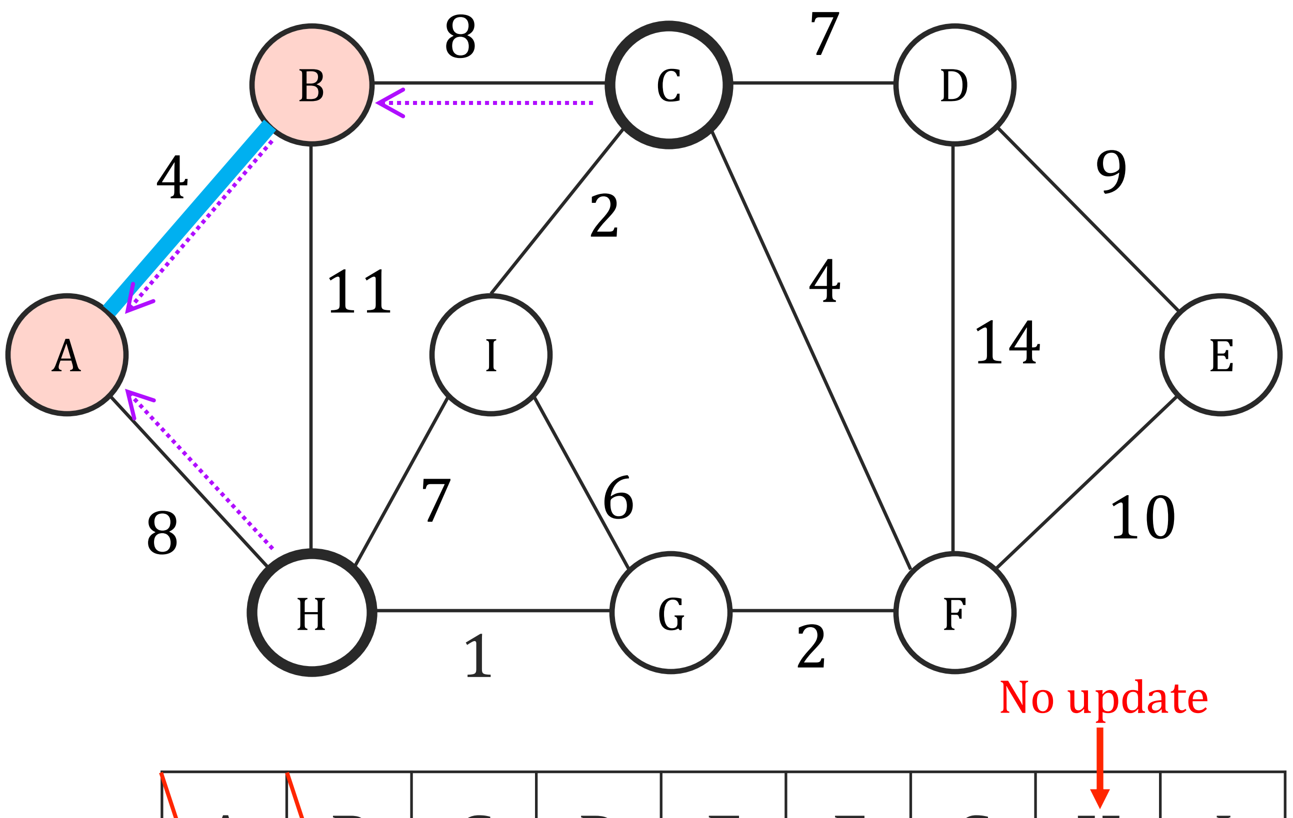
        $prev[z] \leftarrow v$
  return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	**8**	∞	∞	∞	∞	8	∞
prev	∅	*A*	***B***	∅	∅	∅	∅	*A*	∅

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

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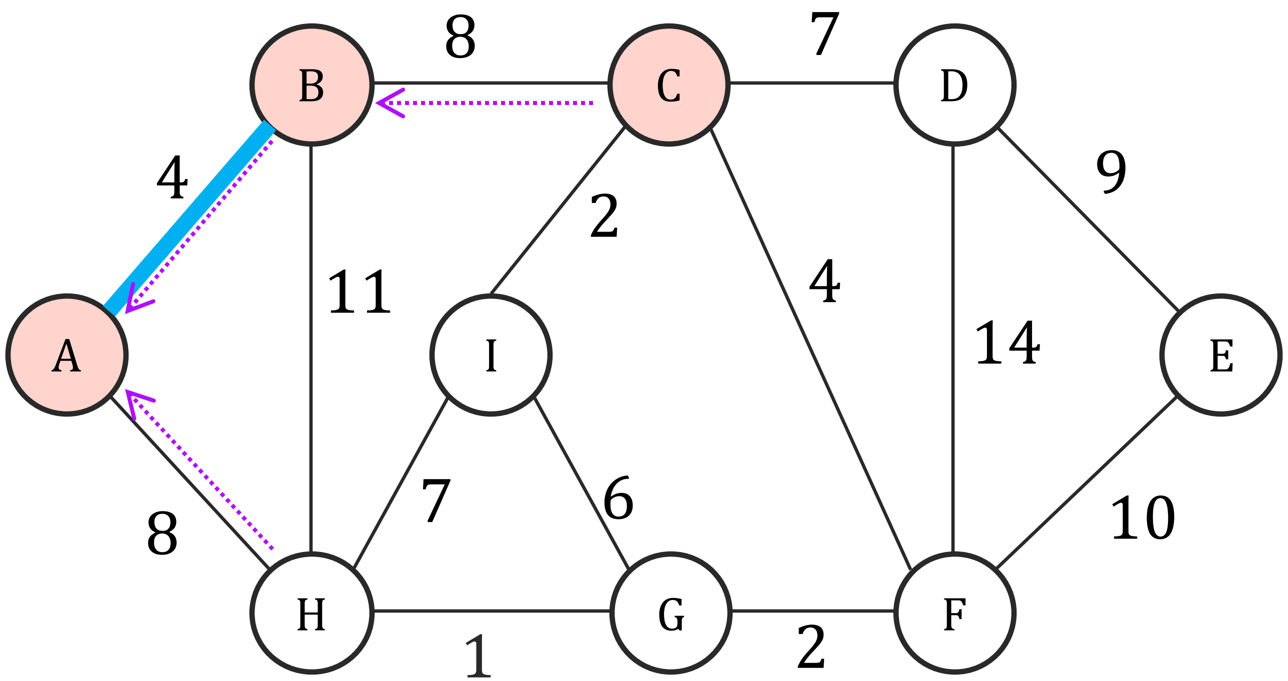
Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	8	∞	∞	∞	∞	8	∞
prev	∅	*A*	*B*	∅	∅	∅	∅	*A*	∅

Fast-Prim$(G = (V, E))$
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  array $prev(n)$ // initialized to null
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  $dist[A] = 0$ // an arbitrary node A
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  while $|X| < |V| - 1$
    $v \leftarrow$ Q.deleteMin
    if $v \neq A$, $X \leftarrow X \cup \{(prev[v], v)\}$
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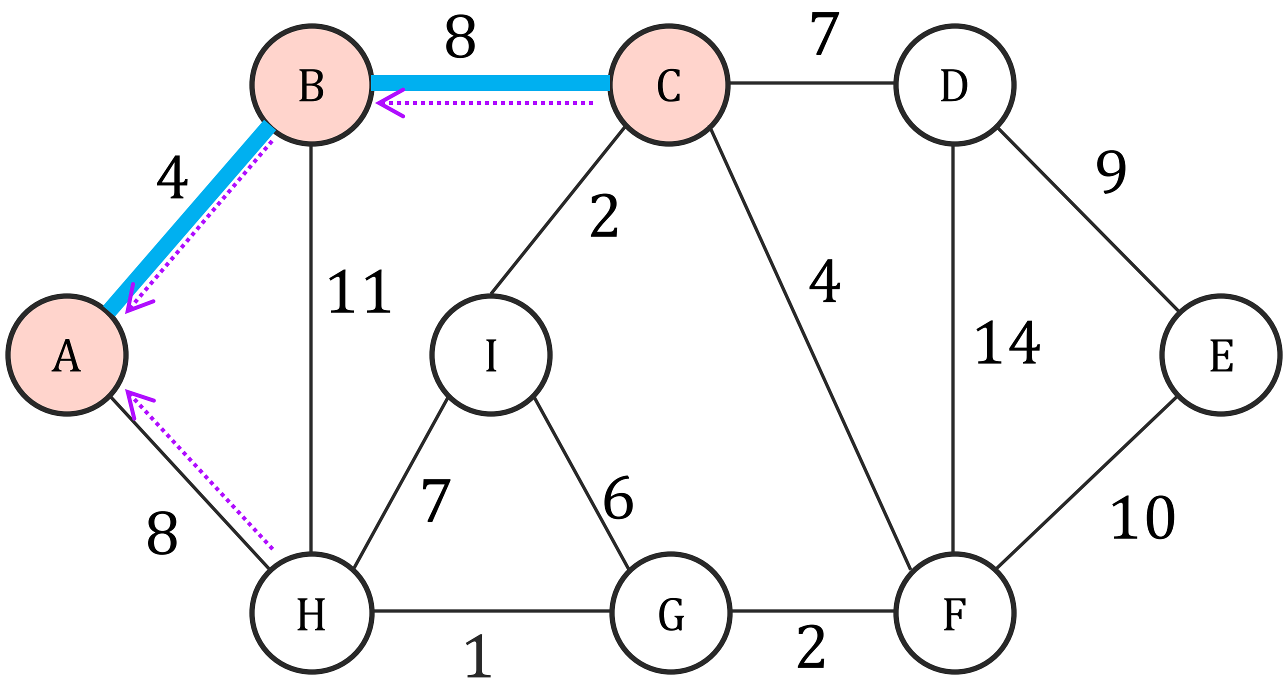
      if $dist[z] > w_{(v,z)}$ and $z \in Q$.
        $Q.\text{decreaseKey}(z, w_{(v,z)})$
        $prev[z] \leftarrow v$
  return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	8	∞	∞	∞	∞	8	∞
prev	∅	*A*	*B*	∅	∅	∅	∅	*A*	∅

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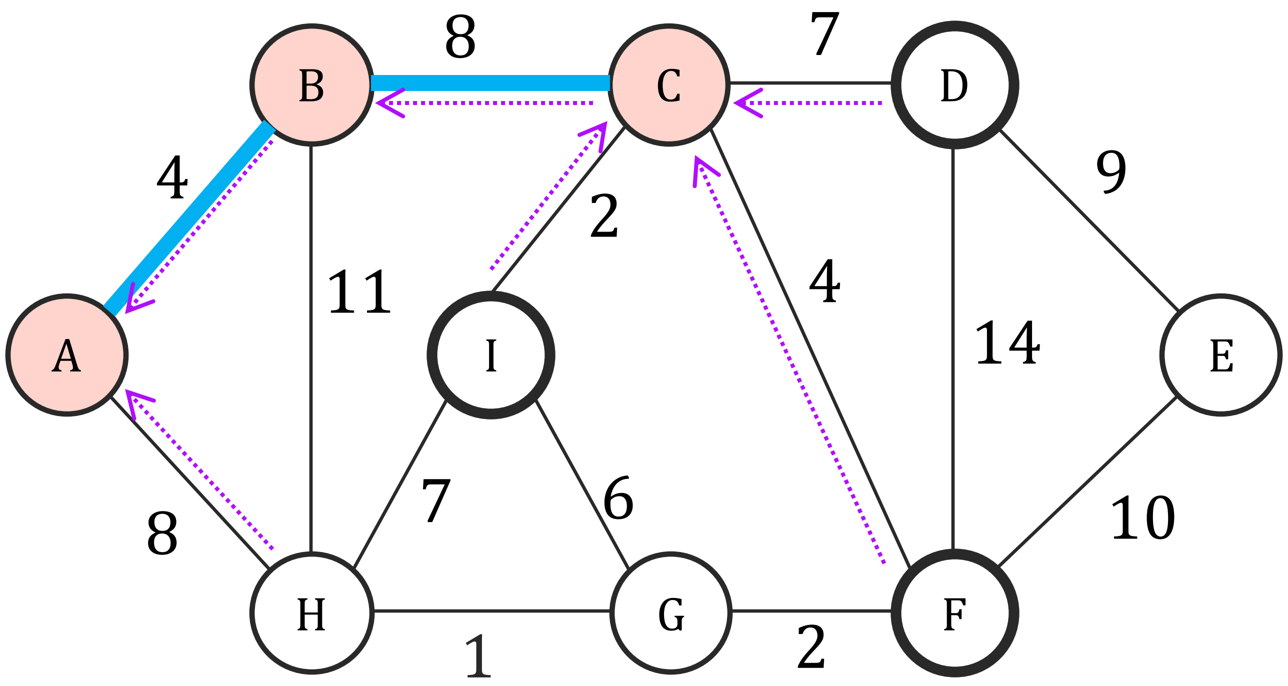
        $v \leftarrow$ Q.deleteMin
        if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$
        for $(v, z) \in E$
            if $dist[z] > w_{(v,z)}$ and $z \in Q$.
                Q.decreaseKey$(z, w_{(v,z)})$
                $prev[z] \leftarrow v$
    return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	8	**7**	∞	**4**	∞	8	**2**
prev	∅	*A*	*B*	***C***	∅	***C***	∅	*A*	***C***

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

array $prev(n)$ // initialized to null

$X = \{\}$ and Q empty priority queue

$dist[A] = 0$ // an arbitrary node A

for $v \in V$, Q.insert$(v, dist[v])$

while $|X| < |V| - 1$

$v \leftarrow$ Q.deleteMin

if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

for $(v, z) \in E$

if $dist[z] > w_{(v,z)}$ and $z \in Q$.

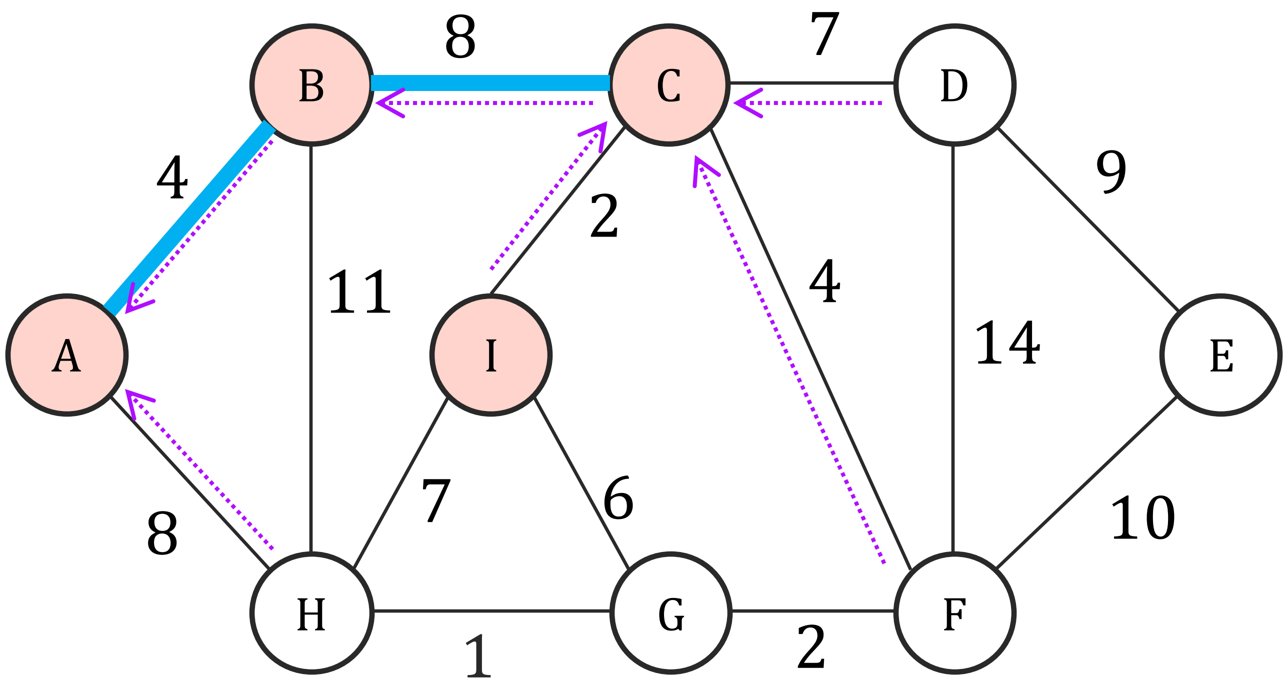
Q.decreaseKey$(z, w_{(v,z)})$

$prev[z] \leftarrow v$

return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	∞	4	∞	8	2
prev	∅	*A*	*B*	*C*	∅	*C*	∅	*A*	*C*

Fast-Prim$(G = (V, E))$

  array $dist(n)$ // initialize to all ∞

  array $prev(n)$ // initialized to null

  $X = \{\}$ and Q empty priority queue

  $dist[A] = 0$ // an arbitrary node A

  for $v \in V$, Q.insert$(v, dist[v])$

  while $|X| < |V| - 1$

    $v \leftarrow$ Q.deleteMin

    if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

    for $(v, z) \in E$

      if $dist[z] > w_{(v,z)}$ and $z \in Q$.

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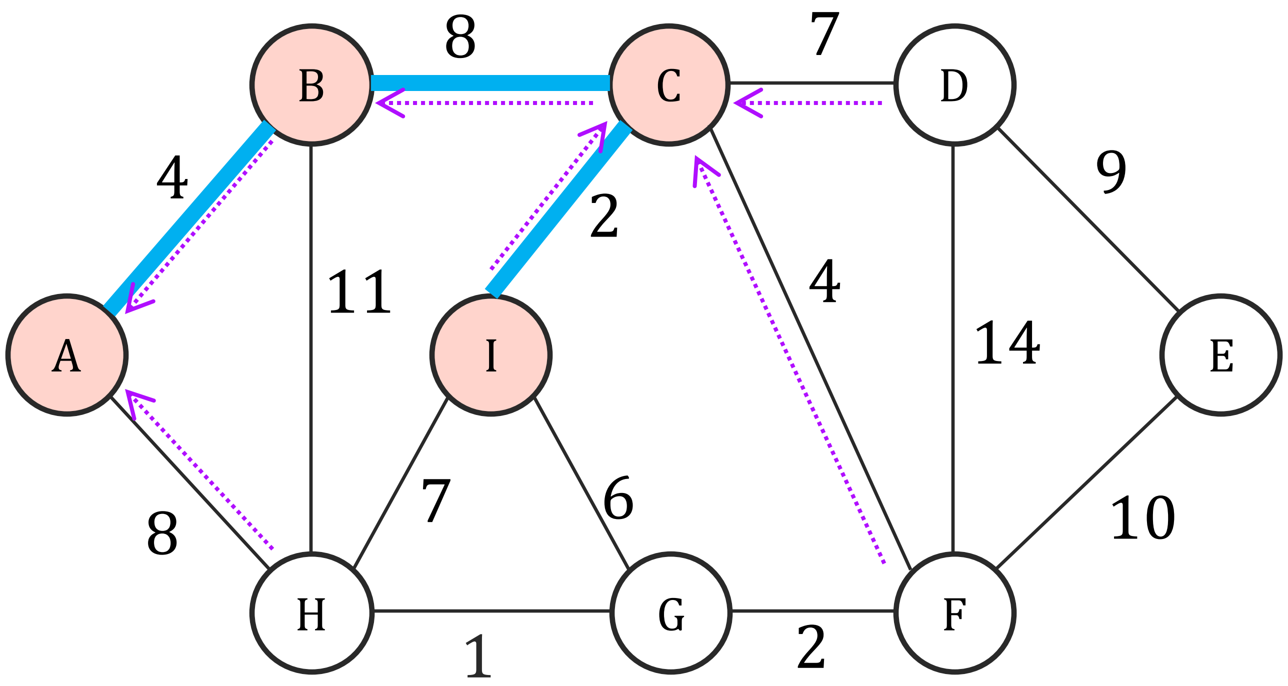
        $prev[z] \leftarrow v$

  return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	∞	4	∞	8	2
prev	∅	*A*	*B*	*C*	∅	*C*	∅	*A*	*C*

Fast-Prim$(G = (V, E))$
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 $dist[A] = 0$ // an arbitrary node A
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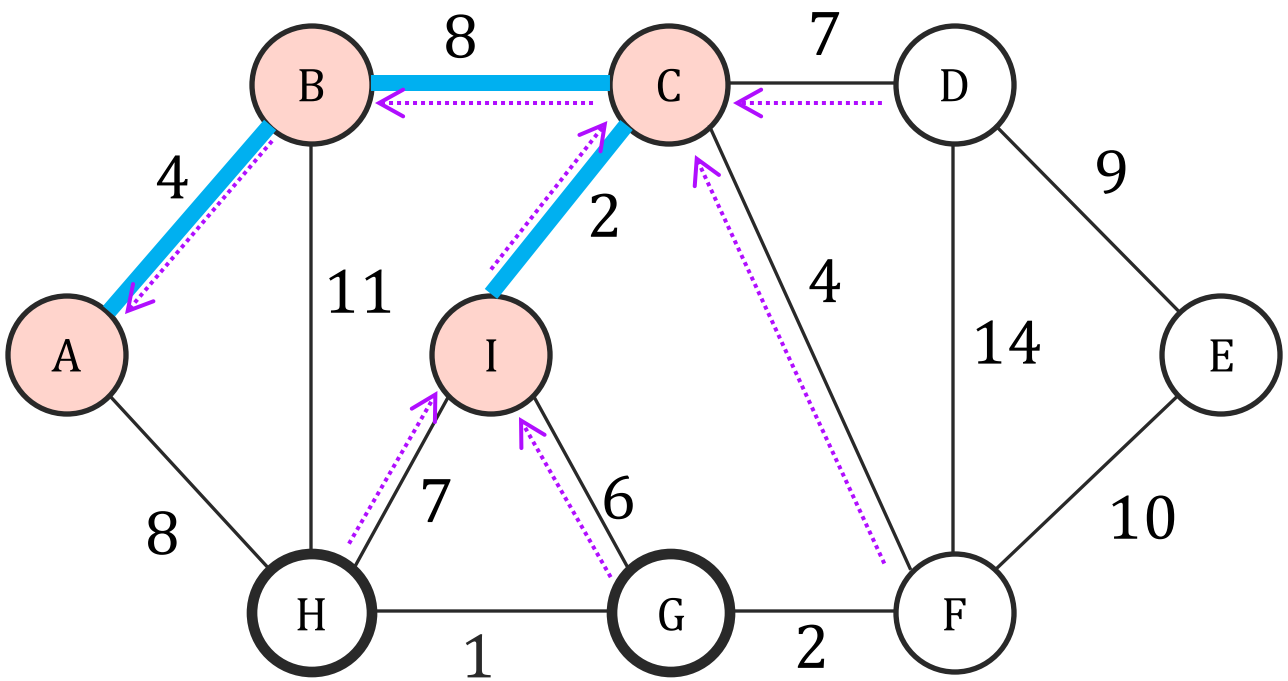
 while $|X| < |V| - 1$
  $v \leftarrow$ Q.deleteMin
  if $v \neq A$, $X \leftarrow X \cup \{(prev[v], v)\}$
  for $(v, z) \in E$
   if $dist[z] > w_{(v,z)}$ and $z \in Q$.
    Q. decreaseKey$(z, w_{(v,z)})$
    $prev[z] \leftarrow v$
 return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	**G**	**H**	I
dist	0	4	8	7	∞	4	**6**	**7**	2
prev	∅	*A*	*B*	*C*	∅	*C*	***I***	***I***	*C*

Fast-Prim$(G = (V, E))$

  array $dist(n)$ // initialize to all ∞

  array $prev(n)$ // initialized to null

  $X = \{\}$ and Q empty priority queue

  $dist[A] = 0$ // an arbitrary node A

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  while $|X| < |V| - 1$

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    if $v \neq A$, $X \leftarrow X \cup \{(prev[v], v)\}$

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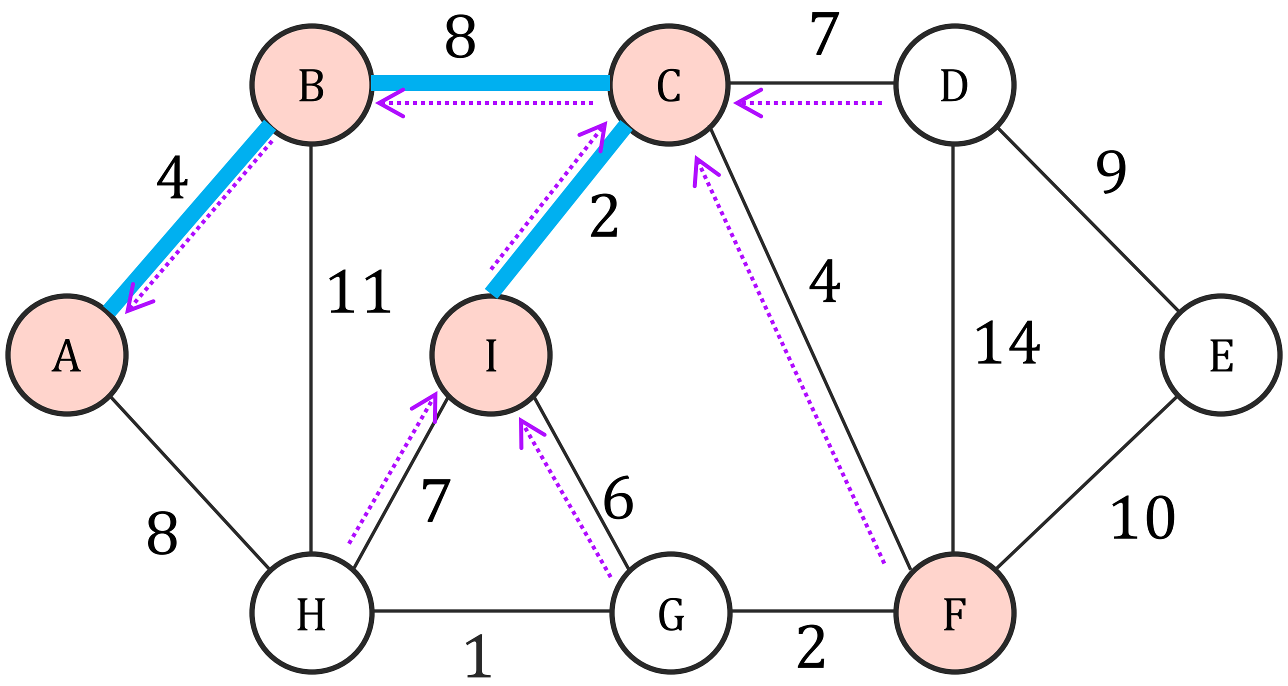
Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*

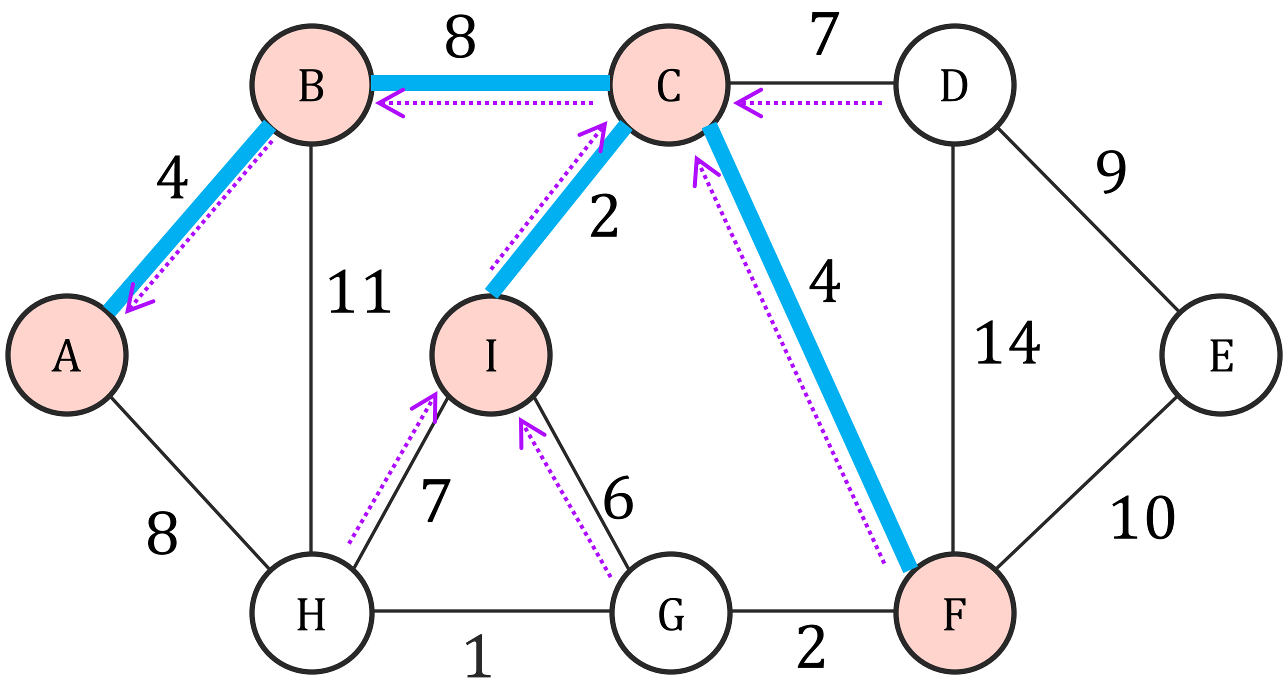


	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	∞	4	6	7	2
prev	Ø	*A*	*B*	*C*	Ø	*C*	*I*	*I*	*C*

Fast-Prim$(G = (V, E))$
  array $dist(n)$ // initialize to all ∞
  array $prev(n)$ // initialized to null
  $X = \{\}$ and Q empty priority queue
  $dist[A] = 0$ // an arbitrary node A
  for $v \in V$, Q. insert$(v, dist[v])$
  while $|X| < |V| - 1$
    $v \leftarrow$ Q.deleteMin
    if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$
    for $(v, z) \in E$
      if $dist[z] > w_{(v,z)}$ and $z \in Q$.
        Q. decreaseKey$(z, w_{(v,z)})$
        $prev[z] \leftarrow v$
  return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	∞	4	6	7	2
prev	∅	*A*	*B*	*C*	∅	*C*	*I*	*I*	*C*

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

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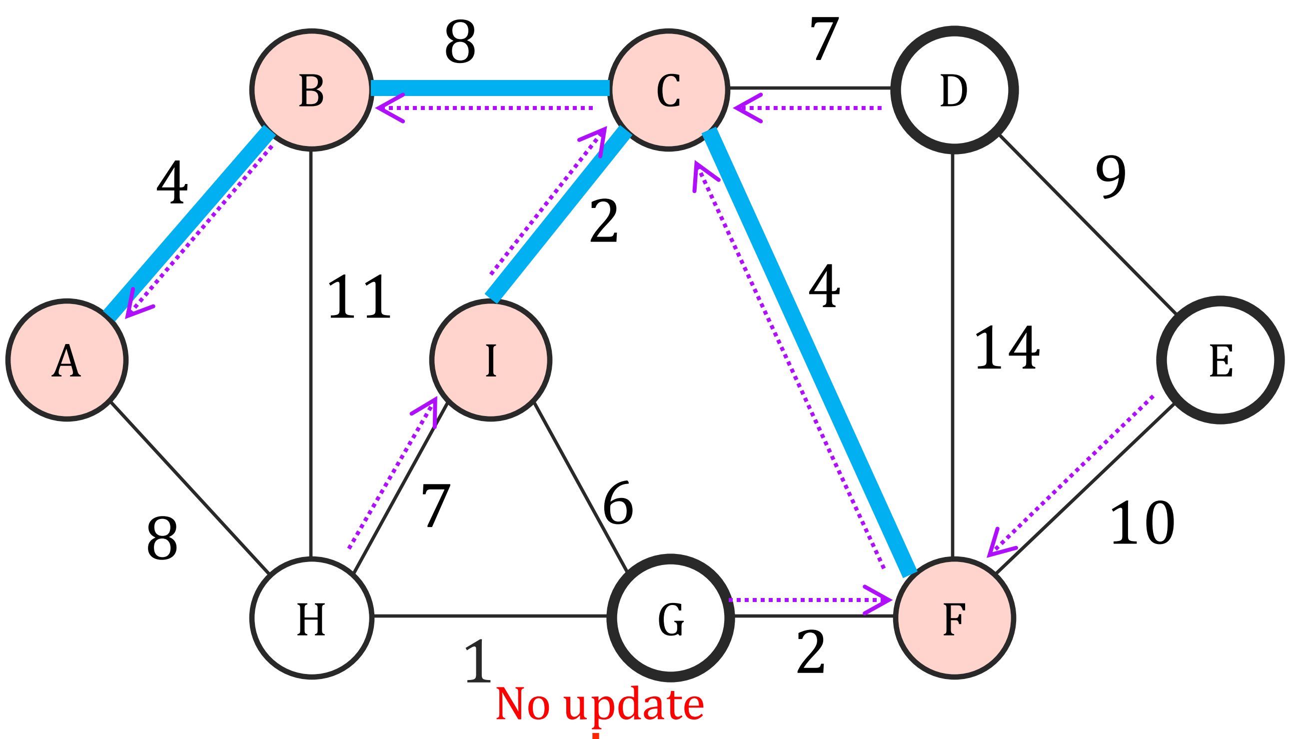


	A	B	C	**D**	**E**	F	**G**	H	I
dist	0	4	8	7	**10**	4	**2**	7	2
prev	∅	*A*	*B*	*C*	***F***	*C*	***F***	*I*	*C*

Fast-Prim$(G = (V, E))$

  array $dist(n)$ // initialize to all ∞

  array $prev(n)$ // initialized to null

  $X = \{\}$ and Q empty priority queue

  $dist[A] = 0$ // an arbitrary node A

  for $v \in V$, Q.insert$(v, dist[v])$

  while $|X| < |V| - 1$

    $v \leftarrow$ Q.deleteMin

    if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

    for $(v, z) \in E$

      if $dist[z] > w_{(v,z)}$ and $z \in Q$.

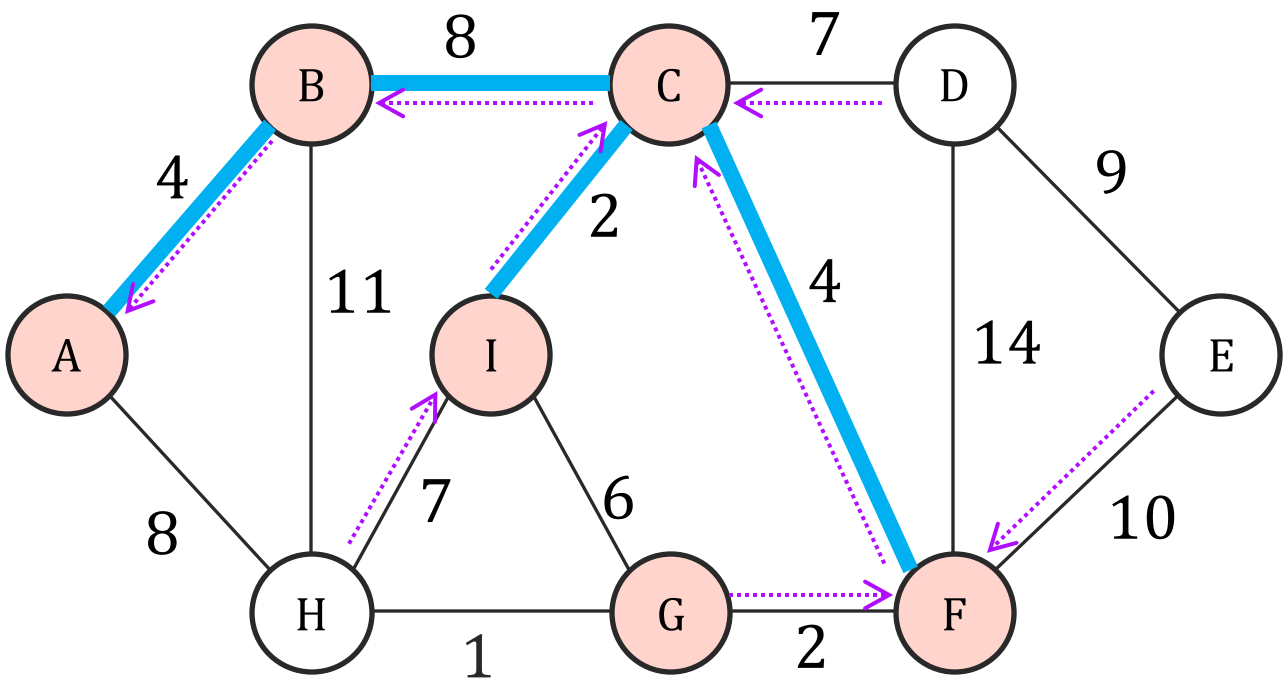
        Q.decreaseKey$(z, w_{(v,z)})$

        $prev[z] \leftarrow v$

  return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q Purple dotted line points to *prev*

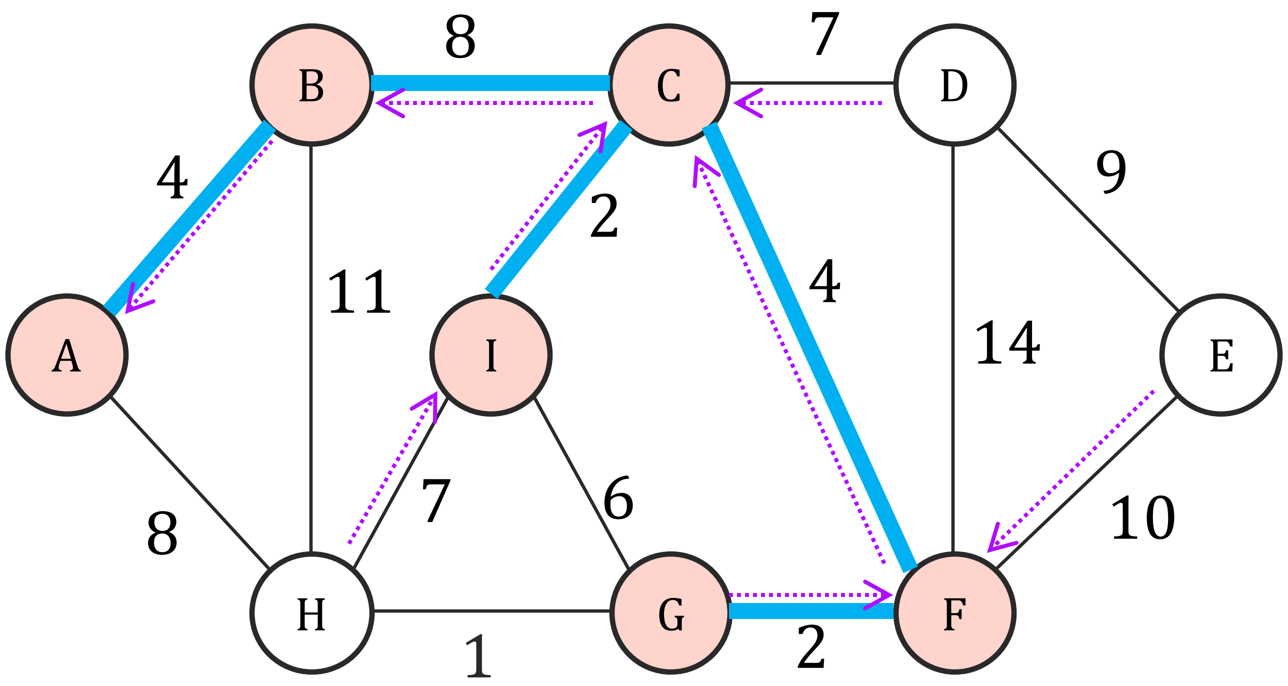


	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	10	4	2	7	2
prev	∅	*A*	*B*	*C*	*F*	*C*	*F*	*I*	*C*

Fast-Prim$(G = (V, E))$
    array $dist(n)$ // initialize to all ∞
    array $prev(n)$ // initialized to null
    $X = \{\}$ and Q empty priority queue
    $dist[A] = 0$ // an arbitrary node A
    for $v \in V$, Q.insert$(v, dist[v])$
    while $|X| < |V| - 1$
        $v \leftarrow$ Q.deleteMin
        if $v \neq A$, $X \leftarrow X \cup \{(prev[v], v)\}$
        for $(v, z) \in E$
            if $dist[z] > w_{(v,z)}$ and $z \in Q$.
                Q.decreaseKey$(z, w_{(v,z)})$
                $prev[z] \leftarrow v$
    return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*

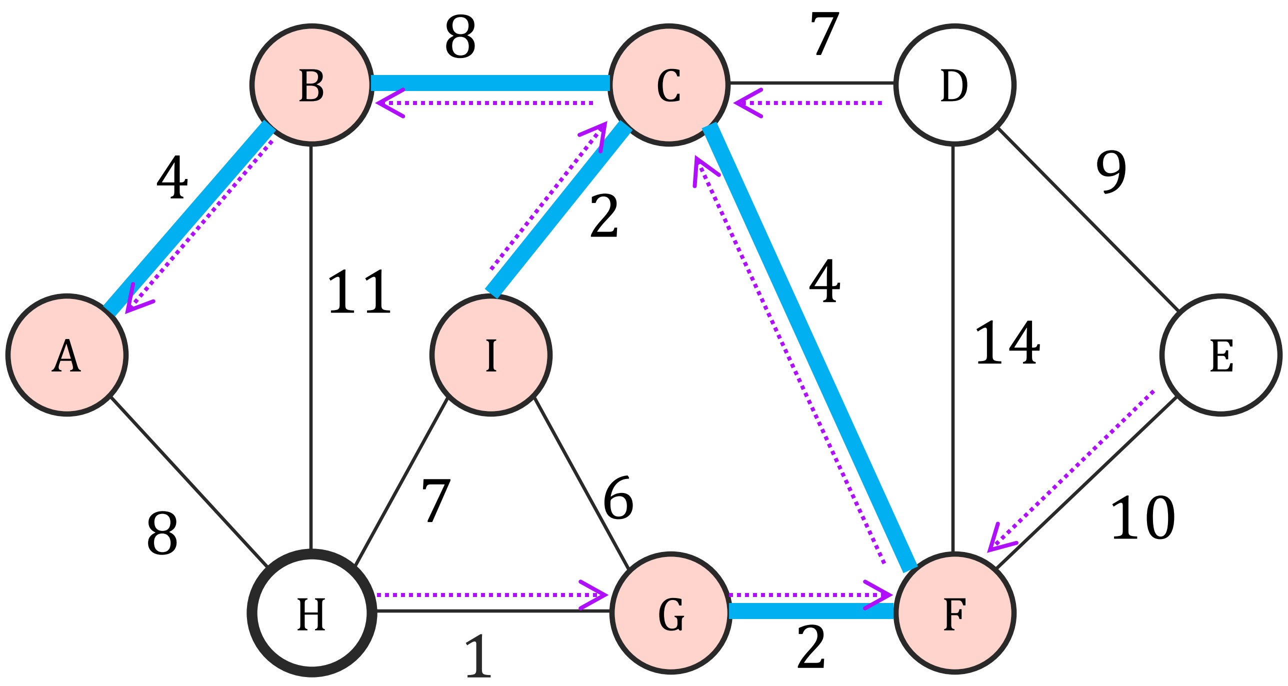


	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	10	4	2	7	2
prev	∅	*A*	*B*	*C*	*F*	*C*	*F*	*I*	*C*

Fast-Prim$(G = (V, E))$
- array $dist(n)$ // initialize to all ∞
- array $prev(n)$ // initialized to null
- $X = \{\}$ and Q empty priority queue
- $dist[A] = 0$ // an arbitrary node A
- **for** $v \in V$, $Q.\text{insert}(v, dist[v])$
- **while** $|X| < |V| - 1$
  - $v \leftarrow$ Q.deleteMin
  - **if** $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$
  - **for** $(v, z) \in E$
    - **if** $dist[z] > w_{(v,z)}$ and $z \in Q$.
      - $Q.\text{decreaseKey}(z, w_{(v,z)})$
      - $prev[z] \leftarrow v$
- **return** X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	**H**	I
dist	0	4	8	7	10	4	2	**1**	2
prev	∅	*A*	*B*	*C*	*F*	*C*	*F*	***G***	*C*

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

array $prev(n)$ // initialized to null

$X = \{\}$ and Q empty priority queue

$dist[A] = 0$ // an arbitrary node A

for $v \in V$, Q. insert$(v, dist[v])$

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$v \leftarrow$ Q.deleteMin

if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

for $(v, z) \in E$

if $dist[z] > w_{(v,z)}$ and $z \in Q$.

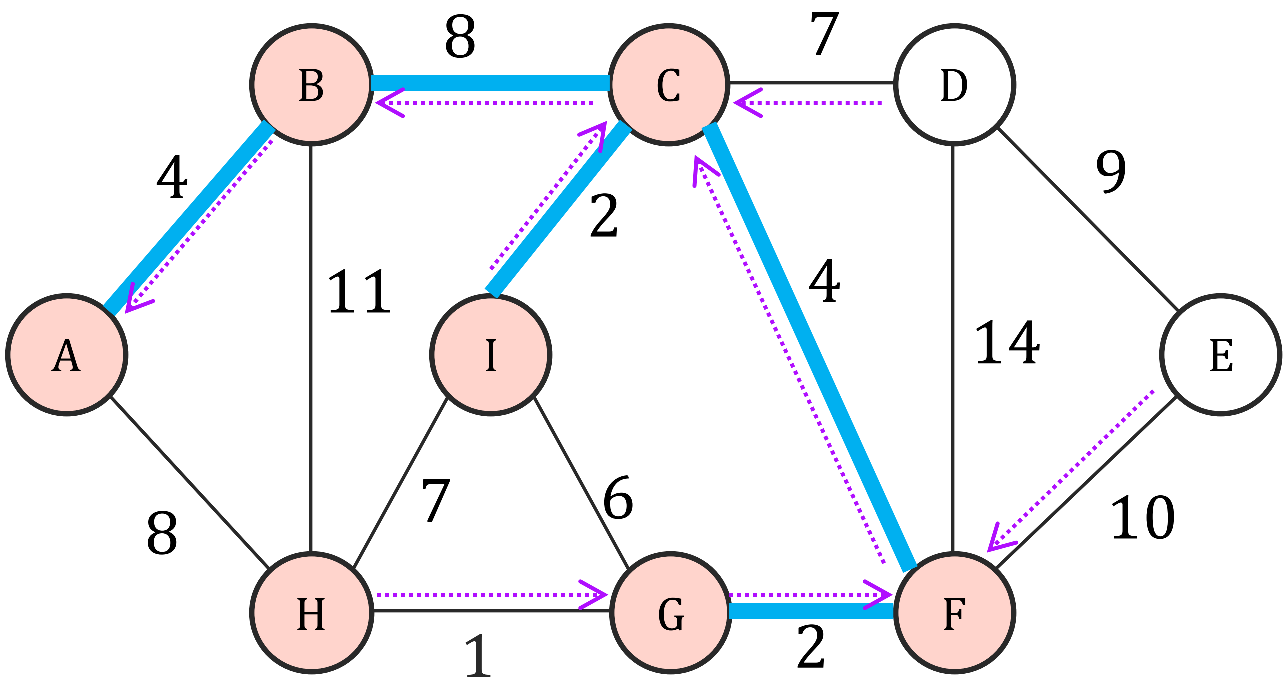
Q. decreaseKey$(z, w_{(v,z)})$

$prev[z] \leftarrow v$

return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	10	4	2	1	2
prev	∅	*A*	*B*	*C*	*F*	*C*	*F*	*G*	*C*

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

array $prev(n)$ // initialized to null

$X = \{\}$ and Q empty priority queue

$dist[A] = 0$ // an arbitrary node A

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if $dist[z] > w_{(v,z)}$ and $z \in Q$.

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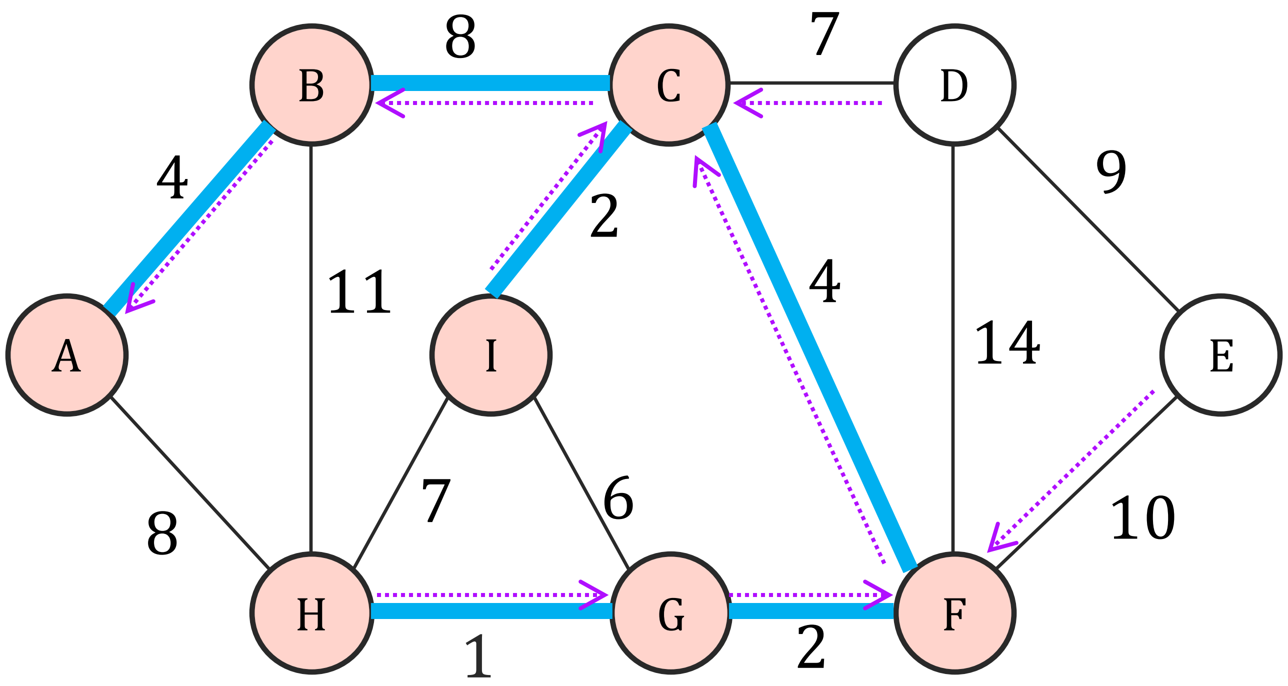
$prev[z] \leftarrow v$

return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*

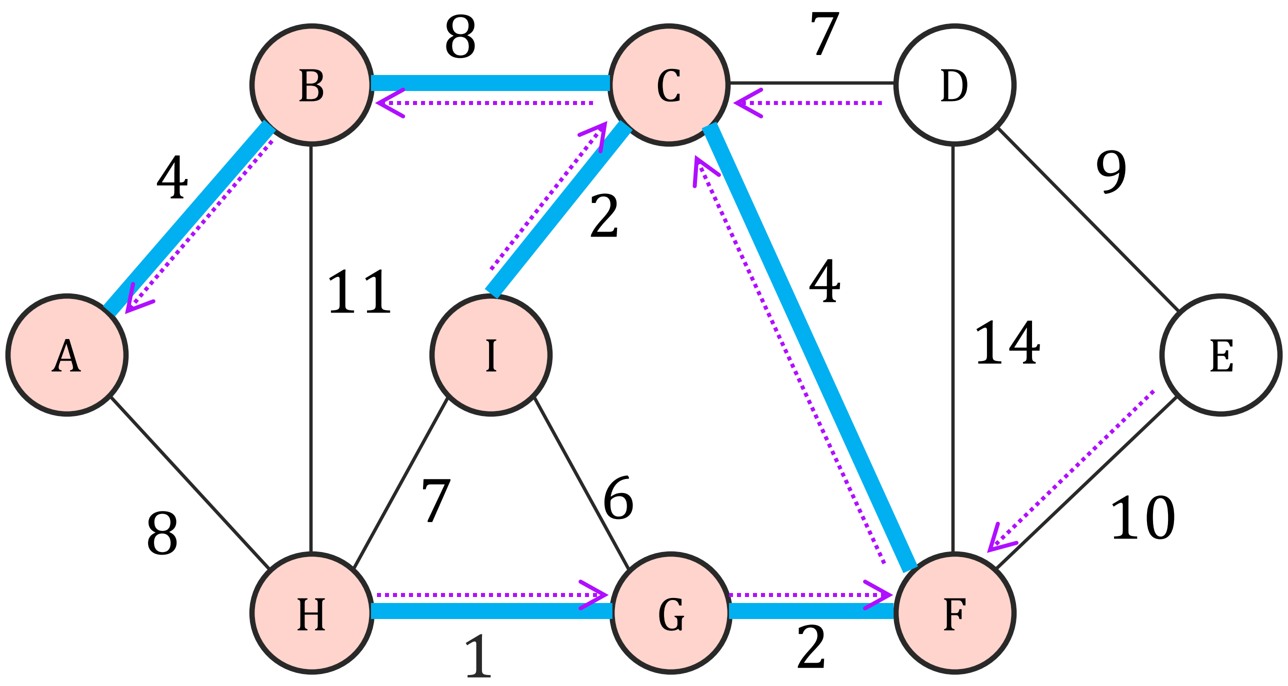


	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	10	4	2	1	2
prev	∅	*A*	*B*	*C*	*F*	*C*	*F*	*G*	*C*

Fast-Prim$(G = (V, E))$
  array *dist*(n) // initialize to all ∞
  array *prev*(n) // initialized to null
  $X = \{\}$ and Q empty priority queue
  $dist[A] = 0$ // an arbitrary node A
  for $v \in V$, Q.insert$(v, dist[v])$
  while $|X| < |V| - 1$
    $v \leftarrow$ Q.deleteMin
    if $v \neq A$, $X \leftarrow X \cup \{(prev[v], v)\}$
    for $(v, z) \in E$
      if $dist[z] > w_{(v,z)}$ and $z \in Q$.
        Q.decreaseKey$(z, w_{(v,z)})$
        $prev[z] \leftarrow v$
  return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q Purple dotted line points to *prev*



Nothing to update

	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	10	4	2	1	2
prev	Ø	*A*	*B*	*C*	*F*	*C*	*F*	*G*	*C*

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

array $prev(n)$ // initialized to null

$X = \{\}$ and Q empty priority queue

$dist[A] = 0$ // an arbitrary node A

for $v \in V$, Q.insert$(v, dist[v])$

while $|X| < |V| - 1$

$v \leftarrow$ Q.deleteMin

if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

for $(v, z) \in E$

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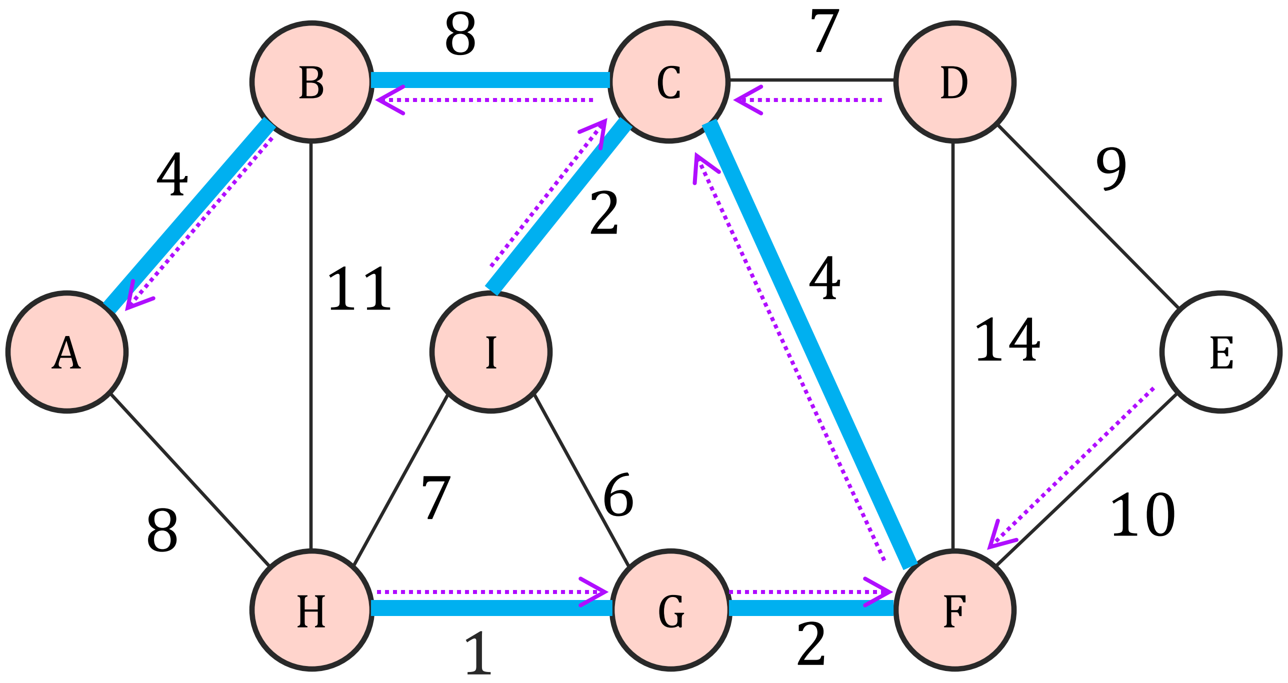
$prev[z] \leftarrow v$

return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*

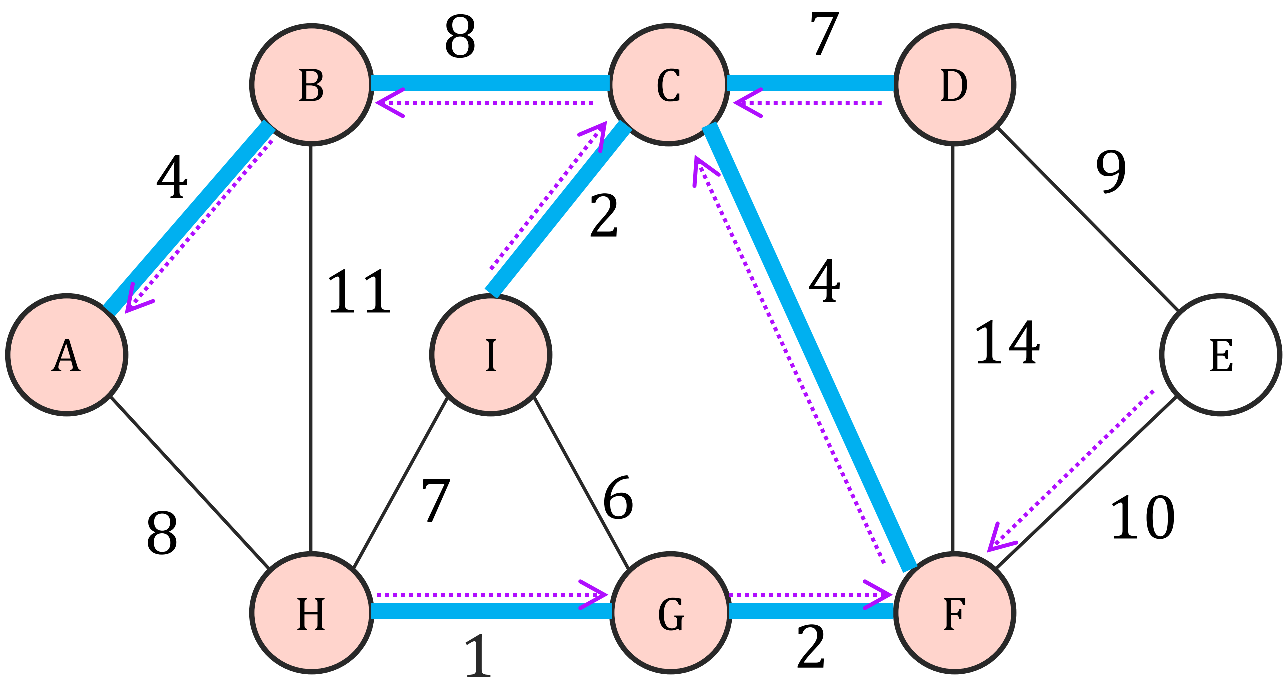


	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	10	4	2	1	2
prev	∅	*A*	*B*	*C*	*F*	*C*	*F*	*G*	*C*

Fast-Prim$(G = (V, E))$
  array $dist(n)$ // initialize to all ∞
  array $prev(n)$ // initialized to null
  $X = \{\}$ and Q empty priority queue
  $dist[A] = 0$ // an arbitrary node A
  for $v \in V$, Q.insert$(v, dist[v])$
  while $|X| < |V| - 1$
    $v \leftarrow$ Q.deleteMin
    if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$
    for $(v, z) \in E$
      if $dist[z] > w_{(v,z)}$ and $z \in Q$.
        Q.decreaseKey$(z, w_{(v,z)})$
        $prev[z] \leftarrow v$
  return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*

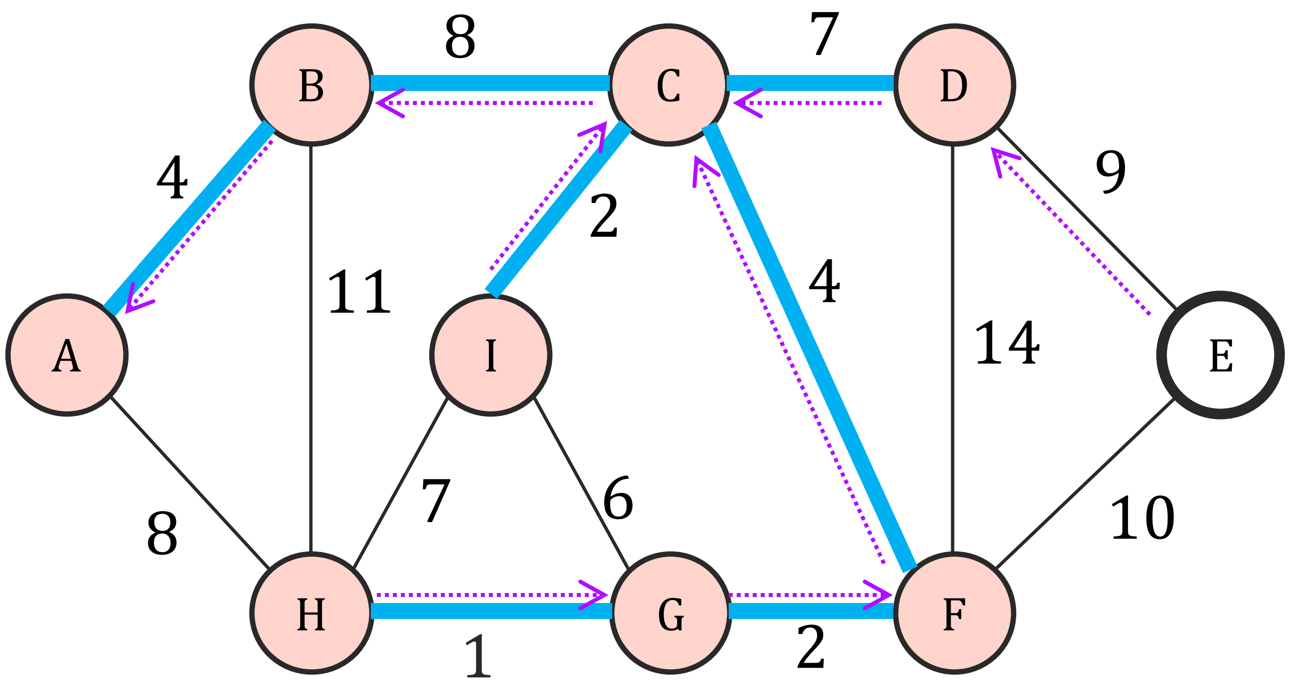


	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	10	4	2	1	2
prev	∅	*A*	*B*	*C*	*F*	*C*	*F*	*G*	*C*

Fast-Prim$(G = (V, E))$
- array $dist(n)$ // initialize to all ∞
- array $prev(n)$ // initialized to null
- $X = \{\}$ and Q empty priority queue
- $dist[A] = 0$ // an arbitrary node A
- **for** $v \in V$, Q.insert$(v, dist[v])$
- **while** $|X| < |V| - 1$
  - $v \leftarrow$ Q.deleteMin
  - **if** $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$
  - **for** $(v, z) \in E$
    - **if** $dist[z] > w_{(v,z)}$ and $z \in Q$.
      - Q.decreaseKey$(z, w_{(v,z)})$
      - $prev[z] \leftarrow v$
- **return** X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	**E**	F	G	H	I
dist	0	4	8	7	**9**	4	2	1	2
prev	∅	*A*	*B*	*C*	***D***	*C*	*F*	*G*	*C*

Fast-Prim$(G = (V, E))$

array $dist(n)$ // initialize to all ∞

array $prev(n)$ // initialized to null

$X = \{\}$ and Q empty priority queue

$dist[A] = 0$ // an arbitrary node A

for $v \in V$, Q.insert$(v, dist[v])$

while $|X| < |V| - 1$

$v \leftarrow$ Q.deleteMin

if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$

for $(v, z) \in E$

if $dist[z] > w_{(v,z)}$ and $z \in Q$.

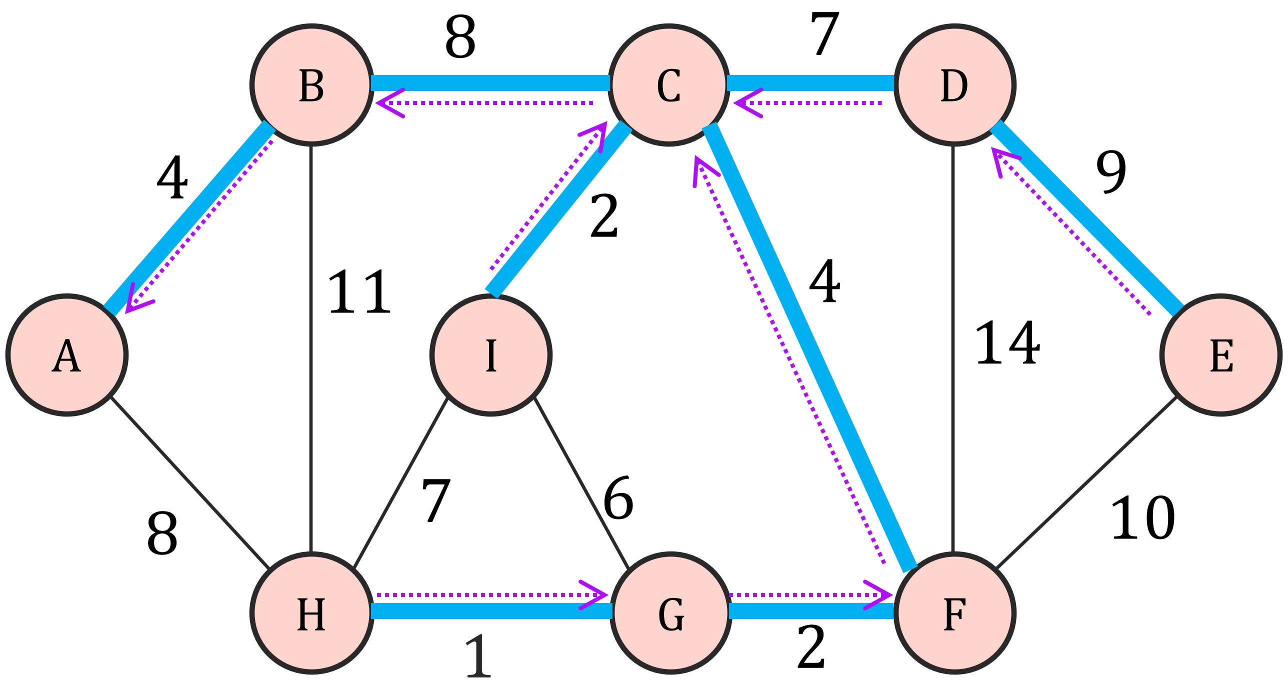
Q.decreaseKey$(z, w_{(v,z)})$

$prev[z] \leftarrow v$

return X

Prim's Algorithm: Efficient Implementation

Red nodes: those deleted from Q   Purple dotted line points to *prev*



	A	B	C	D	E	F	G	H	I
dist	0	4	8	7	9	4	2	1	2
prev	∅	*A*	*B*	*C*	*D*	*C*	*F*	*G*	*C*

```
Fast-Prim(G = (V, E))
    array dist(n)  // initialize to all ∞
    array prev(n)  // initialized to null
    X = {} and Q empty priority queue
    dist[A] = 0 // an arbitrary node A
    for v ∈ V, Q.insert(v, dist[v])
    while |X| < |V| − 1
        v ← Q.deleteMin
        if v ≠ A, X ← X ∪ {(prev[v], v)}
        for (v, z) ∈ E
            if dist[z] > w_(v,z) and z ∈ Q.
                Q.decreaseKey(z, w_(v,z))
                prev[z] ← v
    return X
```

Runtime of Prim's Algorithm

Recall Priority Queue implementations

→Binary heap: $\log(n)$ per operation.

→ Fibonacci Heap: $\log(n)$ for deleteMin, O(1) for insert and decreaseKey.

Runtime of Prim's:

- n Q.inserts
- n Q.deleteMin
- m Q.decreaseKey

With binary heap: $O\big((m+n)\log(n)\big)$.

With Fibonacci heap: $O(m + n\log(n))$

Fast-Prim$(G = (V, E))$

    array $dist(n)$ // initialize to all ∞
    array $prev(n)$ // initialized to null
    X = {} and Q empty priority queue
    $dist[A] = 0$ // an arbitrary node A
    for $v \in V$, Q. insert$(v, dist[v])$
    while $|X| < |V| - 1$
        $v \leftarrow$ Q.deleteMin
        if $v \neq A, X \leftarrow X \cup \{(prev[v], v)\}$
        for $(v, z) \in E$
            if $dist[z] > w_{(v,z)}$ and $z \in Q$.
                Q. decreaseKey$(z, w_{(v,z)})$
                $prev[z] \leftarrow v$
    return X

Comparing MST algorithm's runtimes

- Kruskal's runtime: $O((m+n)\log(n))$
- Prim's runtime: $O(m+n\log(n))$

→ For sparse graphs (m $= O(n)$), both equally good.

→For dense graphs, $(m \gg (n\log(n))$, Prim is much faster than Kruskal.

Other fun facts (no need to memorize):

- $O(m+n)$ expected runtime of a randomized algorithm: Karger, Klein, Tarjan 1995.
- Deterministic $O\big(m\,\alpha(m,n)\big)$: Chazelle 2000

→$\alpha(m,n)$ is called "inverse Ackerman" function and $\alpha(m,n) \leq 5$ for m,n being # of particles in the universe!

- A deterministic algorithm with $O(optimal)$: Pettie, Ramachandran 2002

→ What's "optimal"? No idea!

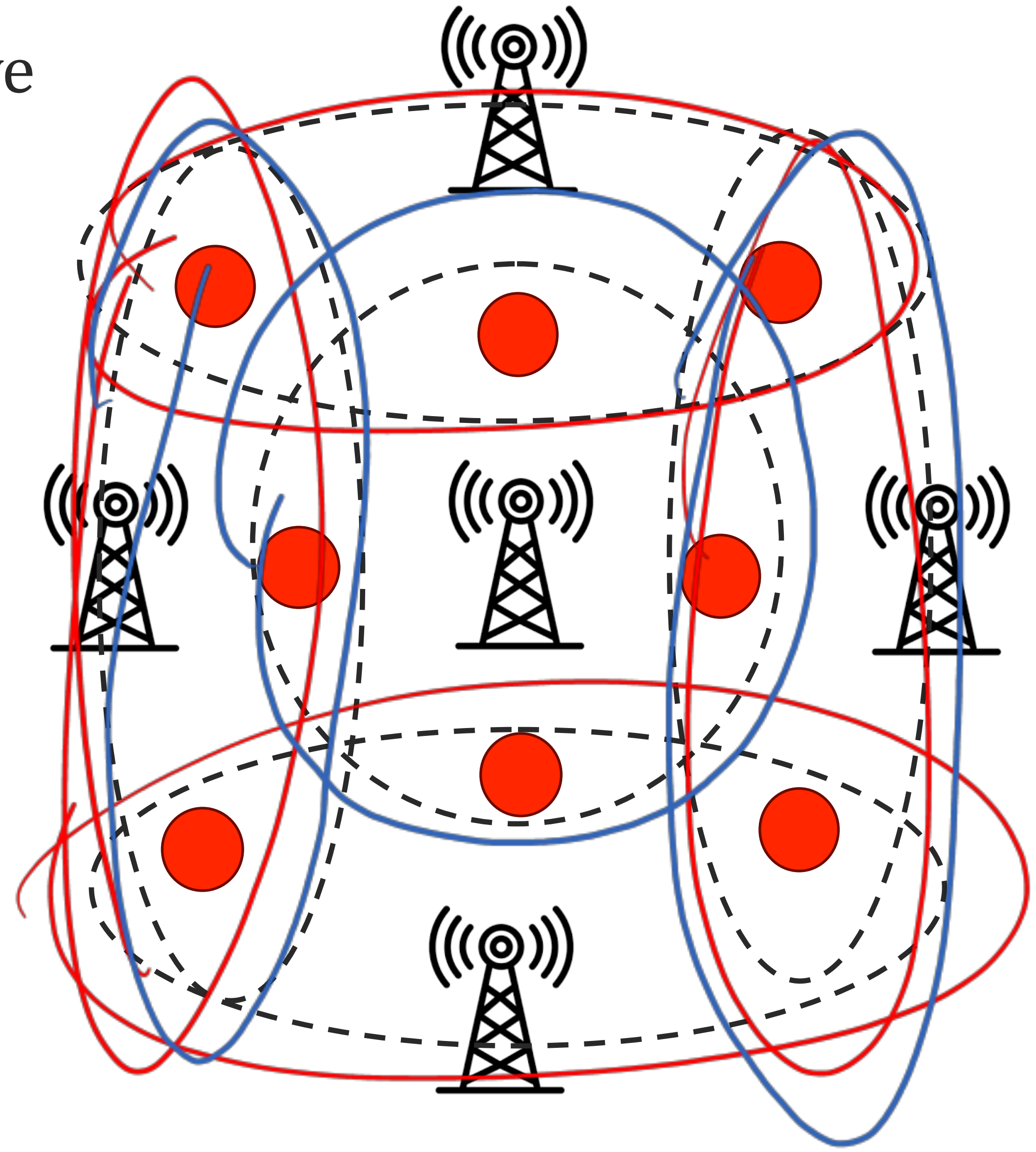
Covering

Imagine, we want to build cell towers so that we provide signal coverage to all houses in a city.

Each **possible location for a cell tower** will cover some homes.

What's the smallest number of cell towers I have to install to cover the city?

Where should these be installed?



The Set Cover Problem

Input:

→Universe of n elements $U = \{1, \ldots, n\}$, and

→Subsets $S_1, S_2, \ldots, S_m \subseteq U$, s.t., $\bigcup_{i=1}^{m} S_i = U$

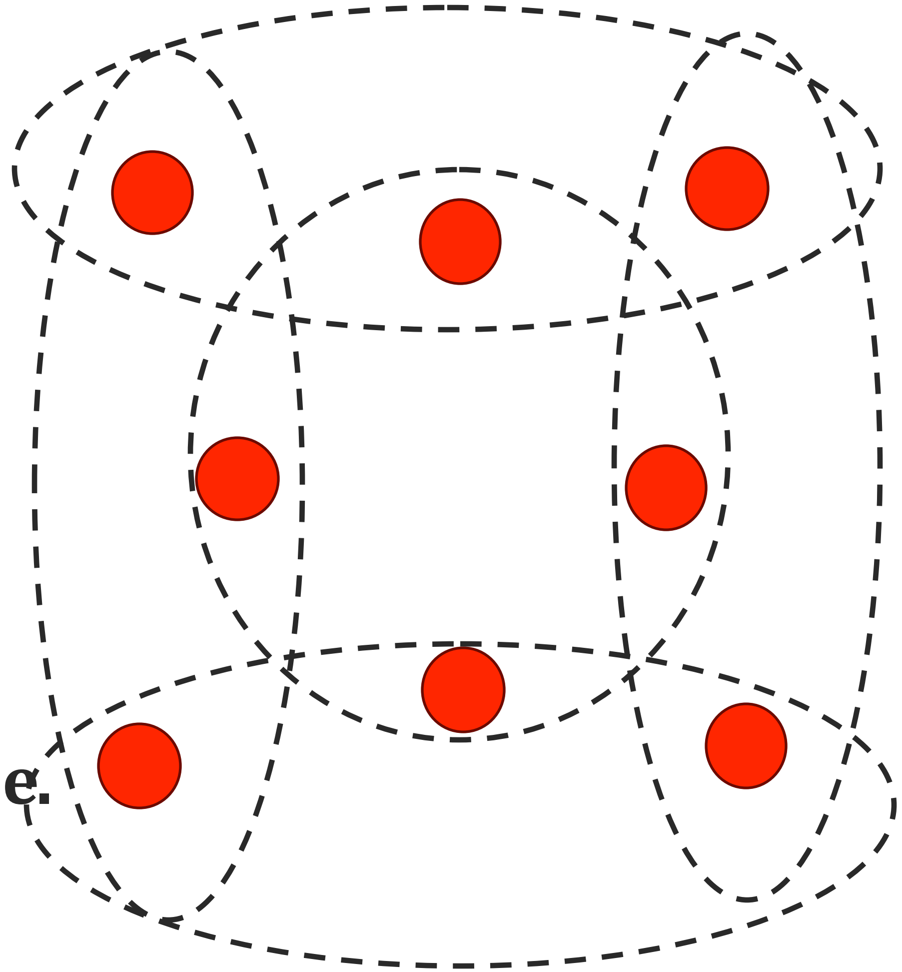
set

{1,3,5} {2,4}

Output:

A collection of subsets covering U of **minimal size.**

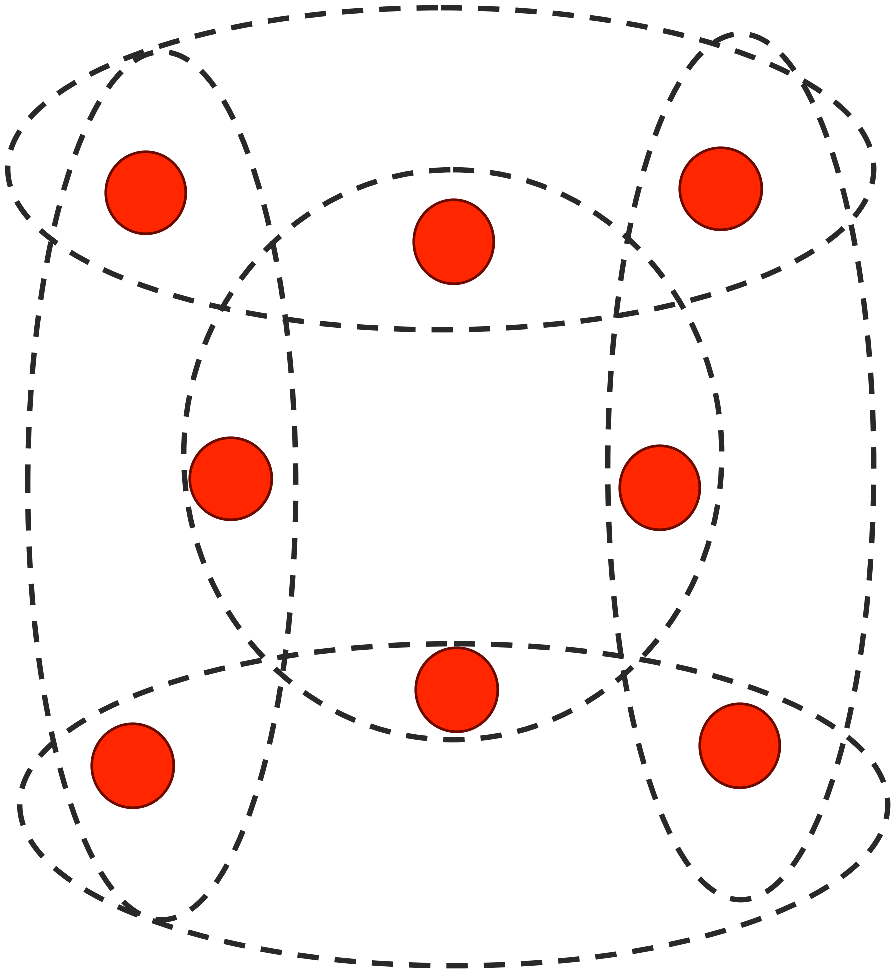
i.e., $J \subseteq \{1,2, \ldots, m\}$ s.t., $\bigcup_{i \in J} S_i = U$



Greedy Algorithm

Discuss

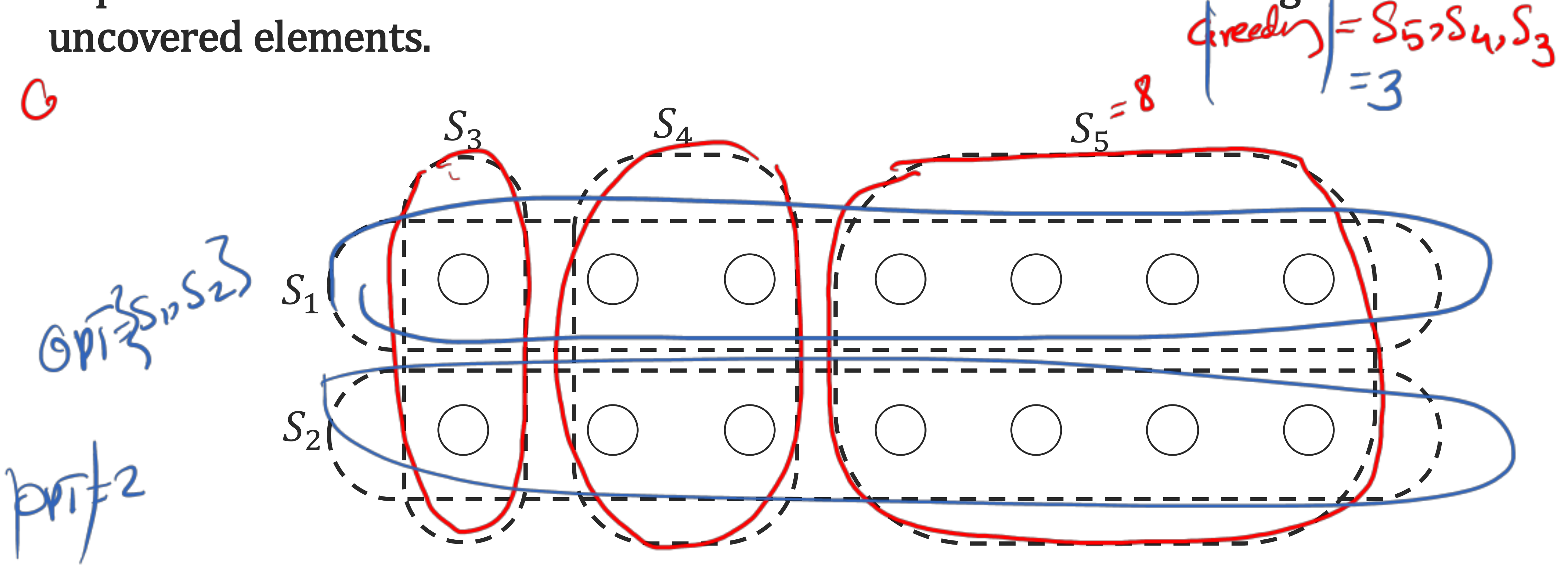
What is a good greedy algorithm?



Greedy Algorithm for Set Cover

A suggested greedy algorithm:

Repeat until all elements of U are covered: Pick the set with the largest number of uncovered elements.



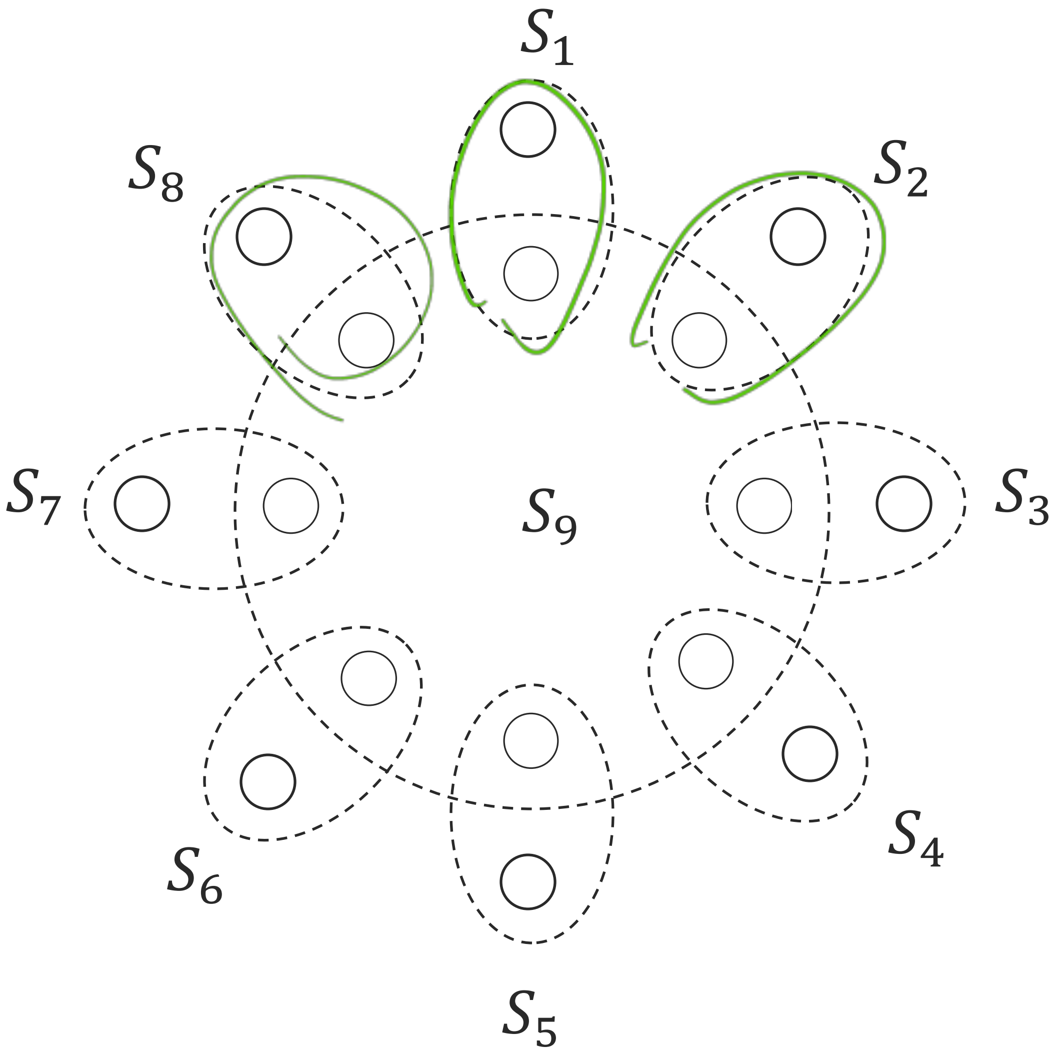
Greedy is not optimal for Set Cover

One other example where this greedy algorithm is not optimal

OPT: $S_1 \dots S_8$

never picking S_9

|OPT| = 8



Greedy: $S_9, S_1, \dots S_8$

|Greedy| = 9

Wrap up

Almost done with being greedy!

→ Just a little left: Greedy is actually reasonably good for set cover.

→ We mastered proof by induction!

→ Scheduling, Minimum Spanning Trees, Horn-Satisfiability, MSTs, Set Cover

Next time

- Set Cover with Greedy
- Dynamic Programming