Continue Linear Programming. Applications.

Lecture in a minute.

What's a linear program?
Variables.
Linear inequalities, and a linear objective function.
Geometrically: a convex region in \( n \).
Optimal solution at “vertex” of region.
Cartoon simplex/duality: move to better vertex, repeatedly.
Applications:
Production Planning.
Variables hiring/firing/inventory/production.
Constraints/Objective encode costs and resource limits.
Bandwidth Problem.
Variables for routes.
Constraints/Objective encode revenue and resource limits.

Linear Programs.
Types of constraints: equality.
Non-negative versus unrestricted.
Standard Form.
Matrix, vector Notation.

Profit maximization.
Plant Carrots or Peas?
2$ bushel of carrots. 4$ for peas.
Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.
100 units of water.
Peas get 3 sq. yards/bushel of sunny land.
Carrots get 3 sq. yards/bushel of shady land.
Garden has 60 sq. yards of sunny land and 75 sq. yards of shady land.

To pea or not to pea, that is the question!

To pea or not to pea.
4$ for peas. 2$ bushel of carrots.
Money \( 4x_1 + 2x_2 \).
Peas take 3 unit of water/bushel.
Carrots take 2 units of water/bushel. 100 units of water.

Optimal Point?

Optimal point?
Try every point if we only had time!
How many points?
Real numbers?
Infinite. Uncountably infinite!

Where's Waldo?

A linear program.

\[
\begin{align*}
\text{max} & \quad 4x_1 + 2x_2 \\
3x_1 & \leq 60 \\
3x_2 & \leq 75 \\
3x_1 + 2x_2 & \leq 100 \\
x_1, x_2 & \geq 0
\end{align*}
\]
Feasible Region.

Vertex is a solution.

Simplex in 2 dimensions.

More variables.

Carpet production planning.

Bandwidth.
Again with carpets!

Production: \( x_i = 20w_i + o_i \)
Employment: \( w_i = w_{i-1} + h_i - f_i \)
Inventory: \( s_i = s_{i-1} + x_i - d_i \)

Regulations: \( o_i \leq 6w_i \)

\[
\begin{align*}
\min & \quad 2000 \sum_i w_i + 320 \sum_i h_i + 400 \sum_i f_i + 8 \sum_i o_i + 180 \sum_i o_i \\
\text{Subject to} & \quad x_i \geq 0 \quad i = 1, 2, \ldots, n
\end{align*}
\]

Diffrernt form!
Not for example: \( x_1 + x_2 \leq 7 \).

Variants of linear programs.

1. Maximization or minimization.
2. Equations or inequalities.
3. Non-negative variables or unrestricted variables.

Peas and carrots.

Standard Form.

Recall Linear equations: \( Ax = b \)?

Can do that here, too!

\[
\begin{align*}
\min & \quad -4x_1 - 2x_2 \\
-2x_1 & \geq -60 \\
-3x_2 & \geq -75 \\
-3x_1 - 2x_2 & \geq -100 \\
x_1, x_2 & \geq 0
\end{align*}
\]

Inputs:
\( m \times n \) matrix \( A \); \( m \) length vector \( b \); \( n \) length vector \( c \).
Output: \( n \) length vector \( x \).

\[
\min cx \\
Ax \geq b
\]

Matrix Form.

\[
\begin{align*}
\min & \quad -4x_1 - 2x_2 \\
-2x_1 & \geq -60 \\
-3x_2 & \geq -75 \\
-3x_1 - 2x_2 & \geq -100 \\
x_1, x_2 & \geq 0
\end{align*}
\]

Inputs: \( m \times n \) matrix \( A \); \( m \) length vector \( b \); \( n \) length vector \( c \).
Output: \( n \) length vector \( x \).

\[
\min cx \\
Ax \geq b
\]

Translations/Reductions.

1. Maximization to minimization?
Multiply objective function by \(-1\).
2. Less than inequalities into greater than?
Multiply both sides by \((-1)\) again! Example: \( 4 \geq 3 \) to \((-1)4 \leq (-1)3\).
3. Inequalities and equalities.

(a) \( \sum a_i x_i \leq b \) into equality?
(b) \( \sum a_i x_i \geq b \) into inequalities?

\[
\sum a_i x_i \leq b \quad \text{and} \quad \sum a_i x_i \geq b
\]

4. Simulate unrestricted variable \( x \) with positive variables.
   - Introduce \( x_+ \) and \( x_- \).
   - Replace \( x \) by \((x_+ - x_-)\).

\((x_+ - x_-)\) could be any real number!

Linear Program Problem

Inputs:
\( m \times n \) matrix \( A \); \( m \) length vector \( b \); \( n \) length vector \( c \).
Output: \( n \) length vector \( x \).

\[
\min cx \\
Ax \geq b
\]

Oh yes, some complexities here.

1. Program has constraints \( x_1 \leq 1 \) and \( x_1 \geq 3 \)?
Has no feasible solution!

Infeasible.

2. Program \( x_1 \geq 0, \max x_1 \).
Optimum?
\( 100, 200, 300 \ldots \) no limit!

Unbounded.
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