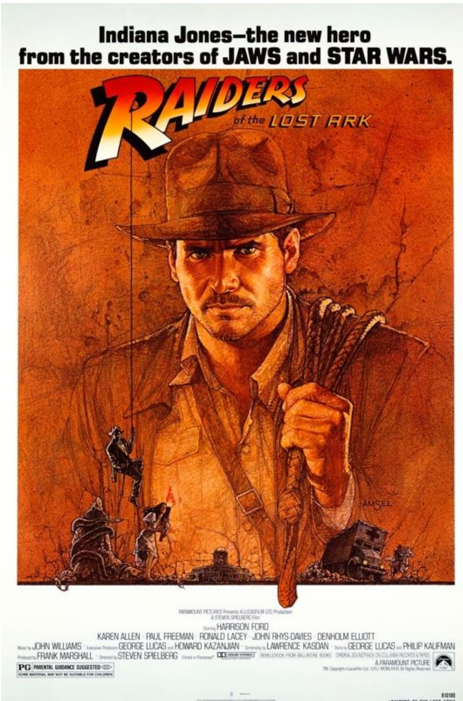
Quantum computing

Quantum computing

Let’s rewind back to **1981**…



IBM 5150 PC

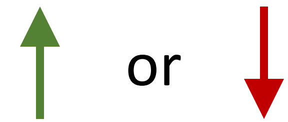
16 kB memory

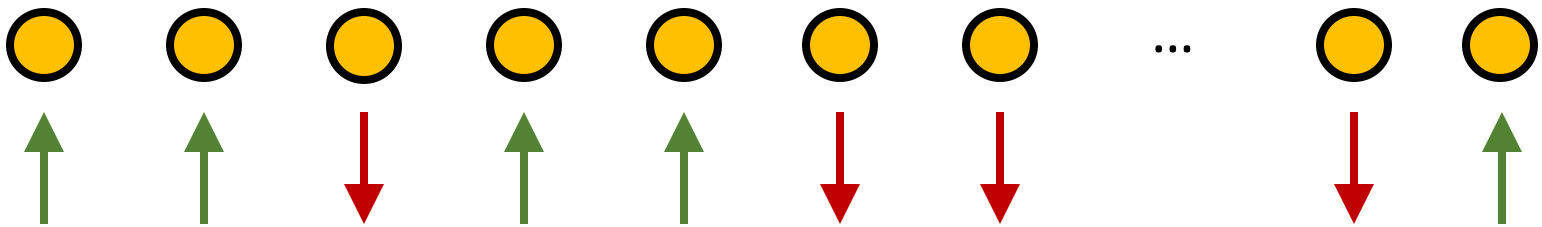
$1,565 *($5,000 today)*

And physicists had a problem:

They couldn’t simulate **quantum systems**

Suppose you have n electrons

Each electron can have a **spin**: 



So that's 2^n possible spin configurations

Quantum Simulation Problem

Given: a starting configuration of the n electrons

Goal: what will the system look like after time T?

Best algorithm for this takes time 2^n!

1981 talk

Richard Feynman

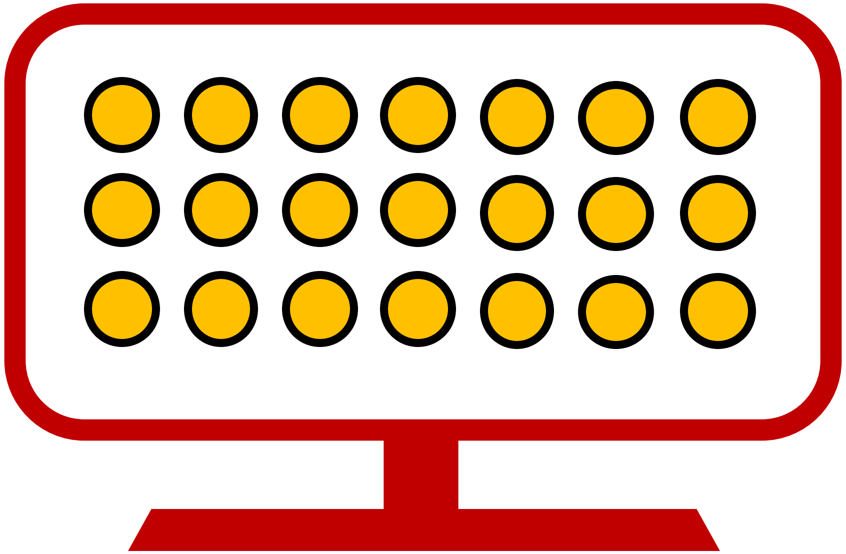
"Nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem because it doesn't look so easy."

"Can you do it with a new kind of computer — a quantum computer? Now it turns out, as far as I can tell, that you can simulate this with a quantum system, with quantum computer elements. It's not a Turing machine, but a machine of a different kind."

Richard Feynman

1981 talk

1. Build a computer out of electrons
2. Make it programmable
3. Let the electrons simulate themselves



After this idea was introduced, people wondered:
how **powerful** are quantum computers?

Sure, they can simulate quantum physics quickly.
But **what else**?

5 algorithms:

1. Deutsch-Jozsa algorithm
2. Bernstein-Vazirani algorithm
3. Simon's algorithm
4. Shor's algorithm
5. Grover's algorithm

I will tell you about **these**

Deutsch-Jozsa problem (1992)

David Deutsch

Richard Jozsa

Input: a function $f: \{0,1\}^n \to \{0,1\}$, either:

1. $f(x) = 0$ for all x
2. $f(x) = 1$ for all x
3. $f(x) = 0$ for half of x, 1 for other half

Goal: which case are we in?

Deterministic algorithms

Have to check $2^{n-1} + 1$ values of $f(x)$.

Quantum algorithm

Solves it in time $O(n)$!

(Why is this a little unsatisfying?)

Bernstein-Vazirani problem (1992)

Ethan Bernstein

Umesh Vazirani

Input: a function $f: \{0,1\}^n \to \{0,1\}$ s.t.

$$f(x) = x_2 + x_5 + \cdots + x_{n-1} \pmod 2$$

(some subset of the input)

Goal: which bits are in the sum?

Classical algorithm:

0. Look at $f(0,0,0,\ldots 0)$
1. Look at $f(1,0,0,\ldots 0)$
2. Look at $f(0,1,0,\ldots 0)$

…

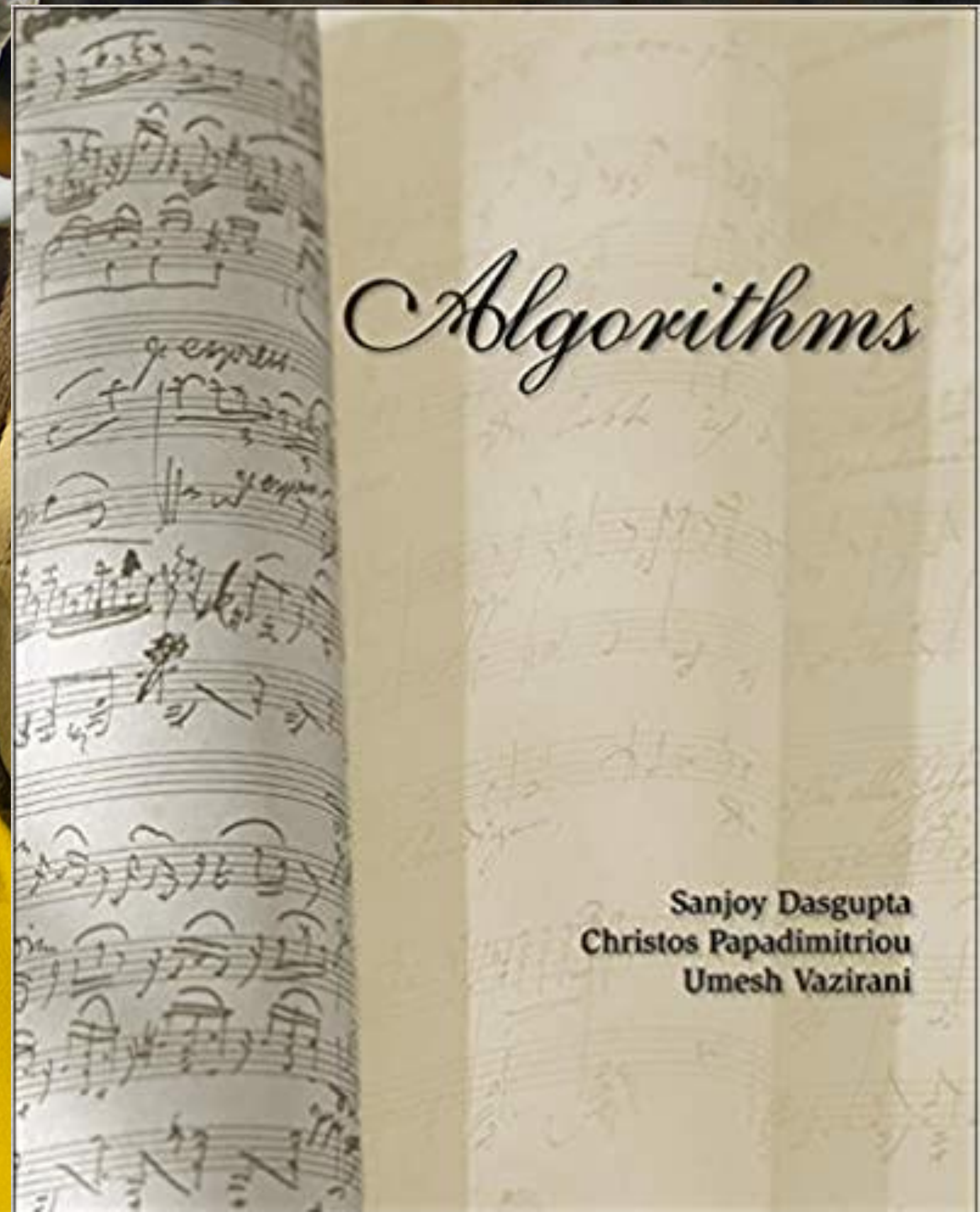
$n+1$ looks at f

Quantum algorithm:

Only need to "quantum look" at f once!

UC Berkeley prof

Umesh Vazirani



Shor's algorithm (1994)

Peter Shor

Lecture 21:

To factor an n-digit number, the best **classical** alg

runs in time $O(e^{1.9 \cdot n^{1/3} \cdot \log(n)^{2/3}})$

Shor's algorithm factors in $O(n^2)$ time

on a **quantum computer**

(also solves discrete log)

With a quantum computer,

we can **break** the RSA cryptosystem!

See: https://www.youtube.com/watch?v=6qD9XElTpCE

Grover's algorithm (1996)

Lov Grover

Lecture 18:

Circuit-SAT is **NP**-complete
Best classical alg runs in $\mathbf{O}(\mathbf{2}^{n})$ time

Grover's algorithm solves **Circuit-SAT**
in $\mathbf{O}\left(\sqrt{\mathbf{2}^{n}}\right) = \mathbf{O}(\mathbf{1.414}^{n})$ time
on a **quantum computer**

(actually solves many other problems
with a **square-root-speedup**)

Quantum speedups

Three types:

1. “**Shor-type speedups**”

    Pro: **Exponentially** faster than classical!
    Con: Only for certain problems (e.g. factoring)

2. “**Grover-type speedups**”

    Pro: Works for many problems!
    Con: Only **polynomially** faster than classical

3. “**Physics simulation speedups**”

    Pro: **Exponentially** faster than classical!
    Con: Only for certain problems (e.g. physics)

Quantum speedups

Q: What is behind these speed-ups?

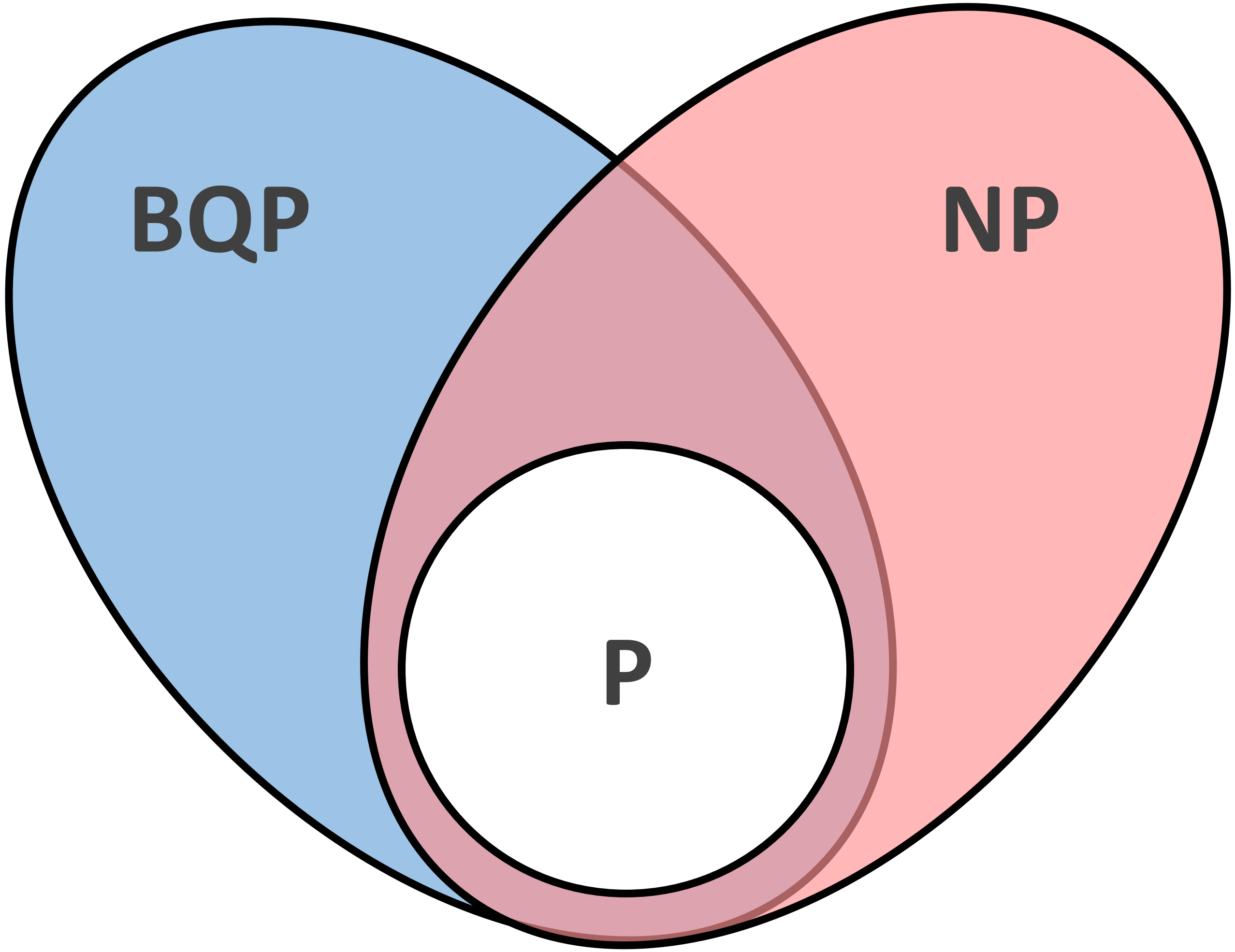
Why do these **outperform** classical algorithms?

1. Deutsch-Jozsa algorithm
2. Bernstein-Vazirani algorithm
3. Simon's algorithm
4. Shor's algorithm
5. Grover's algorithm

A: Quantum computers can do ridiculously fast **Fourier transforms**.

(Recall **lecture 4**: fast Fourier transform helps multiplying polynomials!)

BQP = the set of all problems efficiently solvable on a **quantum computer**



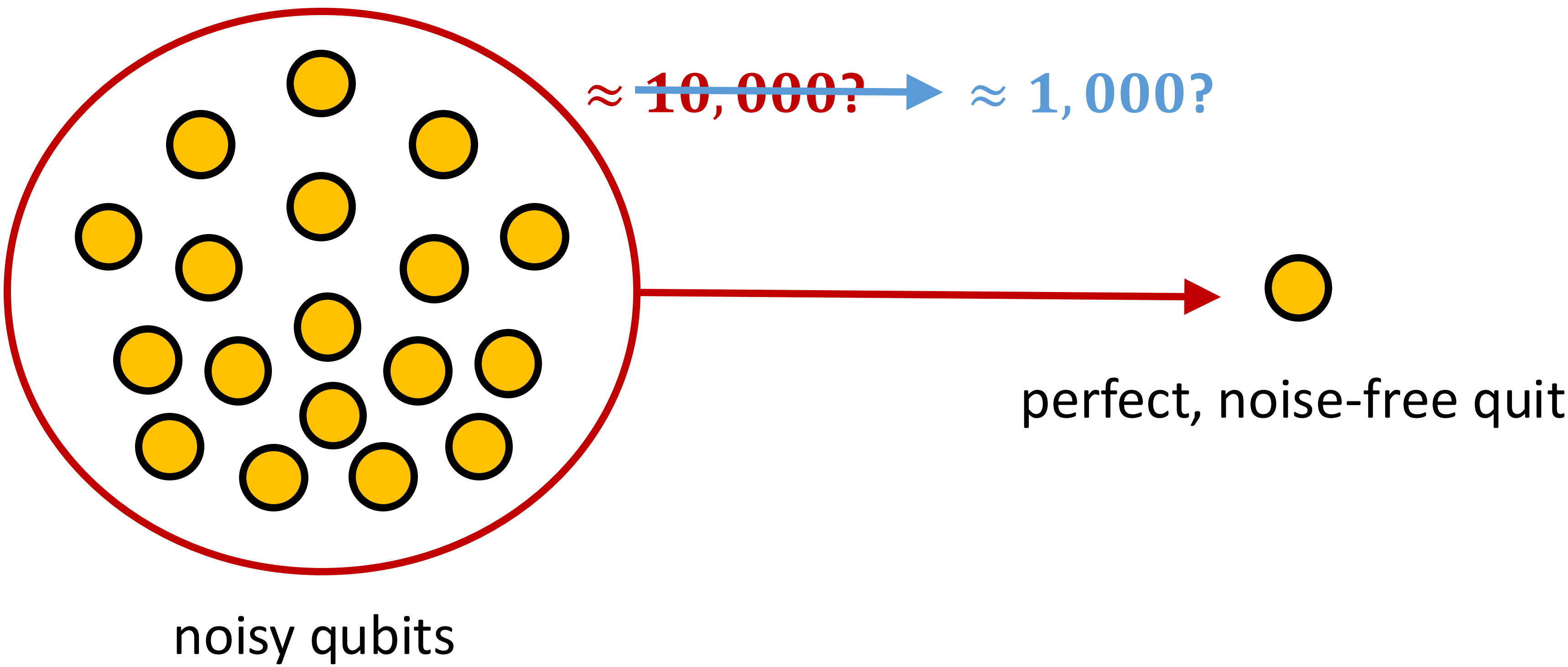
Quantum computers **not believed** to solve **NP**-complete problems efficiently!

Building quantum computers

Building quantum computers is really, really tough!

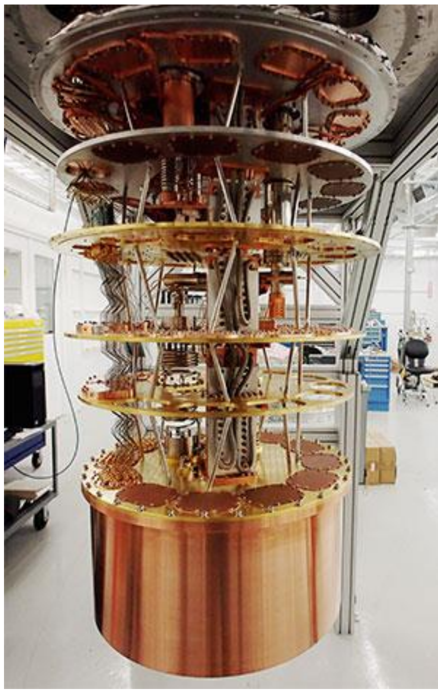
Key challenge: **Noise!** Even the smallest amount of noise completely **ruins** a quantum computation.

Solution: Error correction



2011: Factored $N = 21 = 3$ using two qubits

Today:

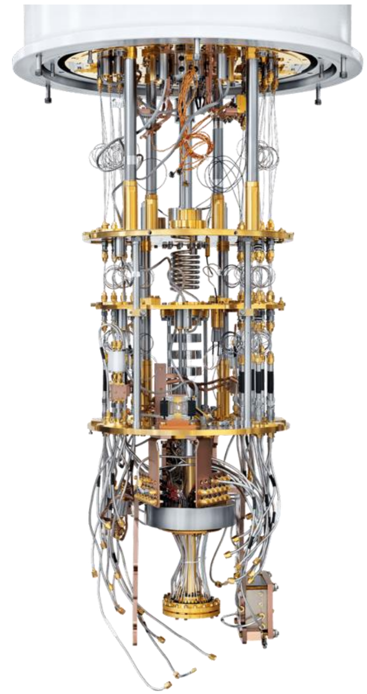


Google

51 qubits

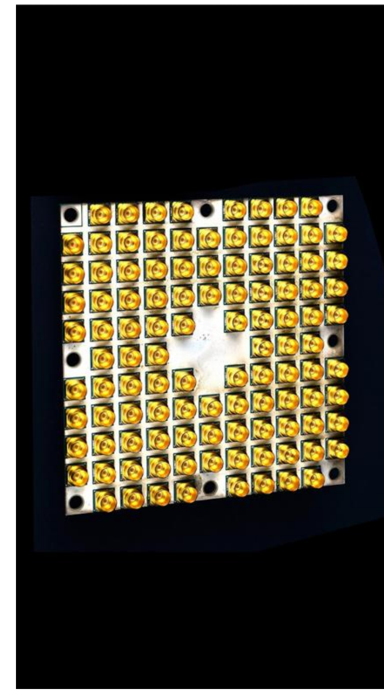
IBM

65 qubits



rigetti

31 qubits



intel

49 qubits

(Noisy qubits. Need 10,000 for 1 clean qubit!)

2011: Factored $N = 21 = 3 \cdot 7$ using two qubits

Today:

How to factor 2048 bit RSA integers with less than a million noisy qubits

Craig Gidney

Google Quantum AI, Santa Barbara, California 93117, USA

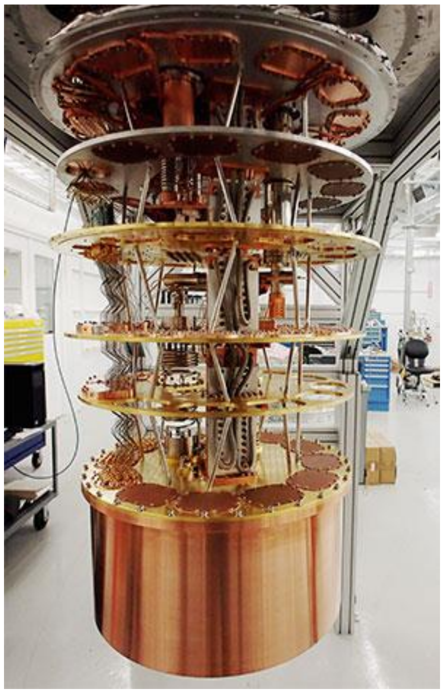
June 9, 2025

Google	IBM	rigetti	intel
51 qubits	65 qubits	31 qubits	49 qubits

(Noisy qubits. Need 1,000 for 1 clean qubit!)

2011: Factored $N = 21 = 3$ using two qubits

Today:



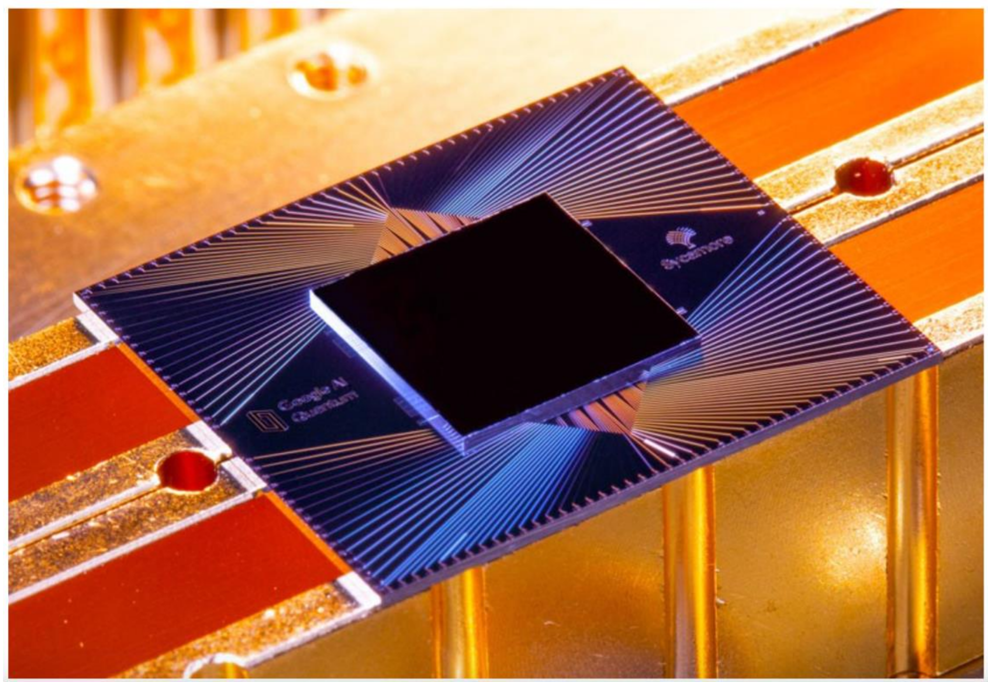
Google

51 qubits

Hello quantum world! Google publishes landmark quantum supremacy claim

The company says that its quantum computer is the first to perform a calculation that would be practically impossible for a classical machine.

Elizabeth Gibney



Now let’s see
a quantum algorithm