CS 170: Algorithms

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Hello and ...

H . . . H . S . H H

CS 170: Algorithms

Hello and ...

H . . . H .   . H H
S

CS 170: Algorithms

Hello and ...

H . . . H .   . H

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CS 170: Algorithms

Hello and ...

H . . . . . H

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Hello and ...

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CS 170: Algorithms

Hello and ...

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CS 170: Algorithms

Hello and ...

S H H H H .

Hello and ...

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CS 170: Algorithms

Hello and ...

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CS 170: Algorithms

Hello and ...

S H H H H . . . .

CS 170: Algorithms

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S H H H H . . . . . .

CS 170: Algorithms

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S H H H H . . . . . .

Please,

CS 170: Algorithms

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S H H H H . . . . . .

Please, limit laptops (unless lecture draft slides),

CS 170: Algorithms

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S H H H H . . . . . .

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Thank you!

Lecture in a minute.

MergeSort.

Lecture in a minute.

MergeSort.
  Sort two halves, put together.
    Merge: two pointer scan.
  $T(n) = 2T(\frac{n}{2}) + O(n)$.
  $T(n) = O(n \log n)$.
Also: iterative view.

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Median finding.
- Selection: more general, “strengthen induction.”
- Random pivot element to split elements.
- Recurse on one subset.

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Expected Time Analysis:
$O(n)$ time to decrease size by $3/4$.

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Iterative Mergesort: Bottom up, use of queues.

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Make each element into list and put lists in queue.

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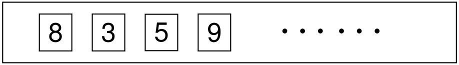

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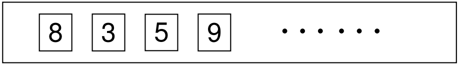
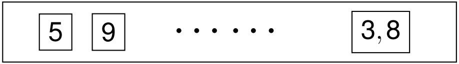


Merge first two lists, put in queue (at end).



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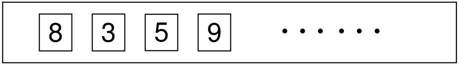

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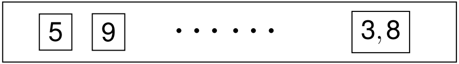

Rinse.

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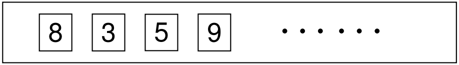

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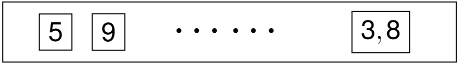

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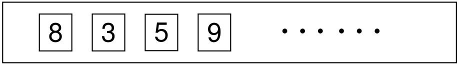

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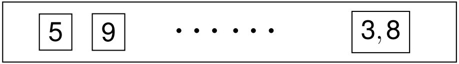

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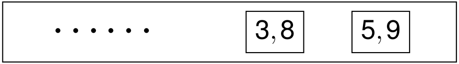

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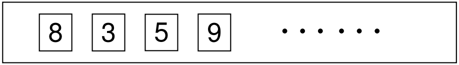

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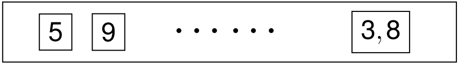

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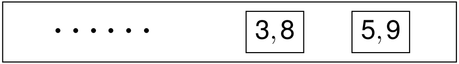

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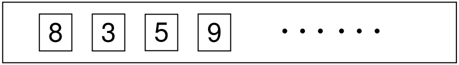

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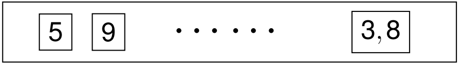

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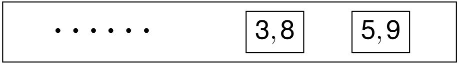

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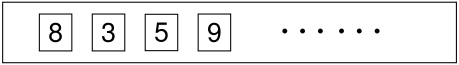

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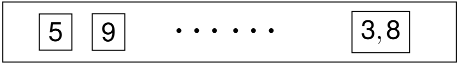

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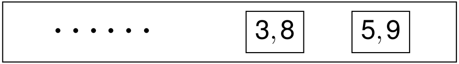

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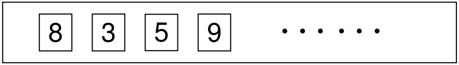

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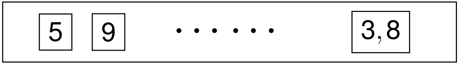

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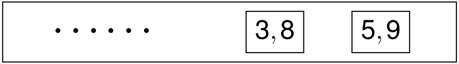

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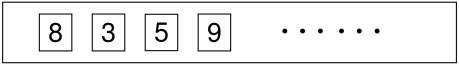

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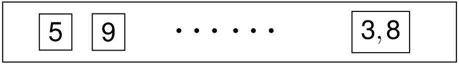

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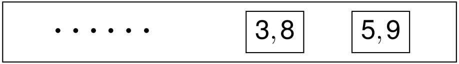

And next pass through queue...

Each pass through queue: each element touched once. $O(n)$ time.

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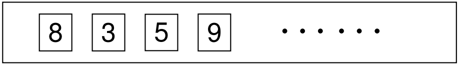

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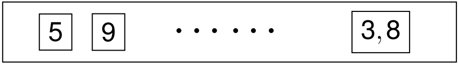

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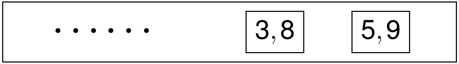

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Each pass halves number of lists.

Check it out...

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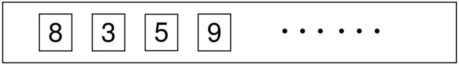

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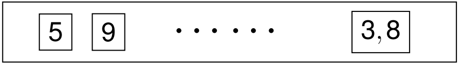

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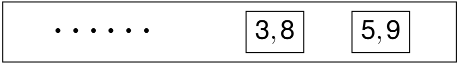

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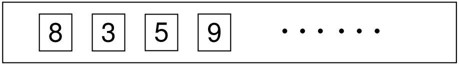

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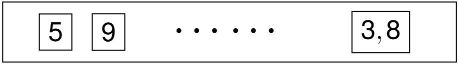

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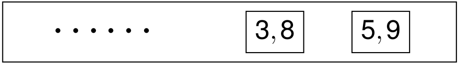

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Sorting lower bound.

Can we do better?

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Comparison sorting algorithm only compares numbers.

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Does not look at bits only uses result of comparison.

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Merge:
Compare two first elts and then output first.

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Comparison sort?

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“Radix” Sort.

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Bucket according to whether begins with "A", "B"....

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Looks at characters...

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Repeat in each bucket with next characters.
Looks at characters... or looks at “bits”.
Not a comparision sort.!

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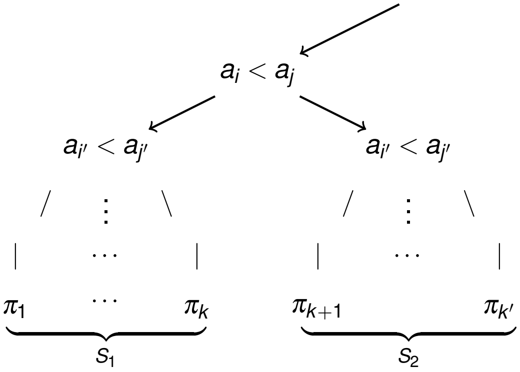
Figure for proof.



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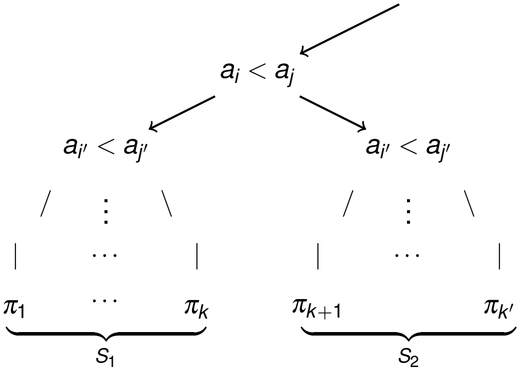

Either the set of permutations S_1 or S_2 is larger.

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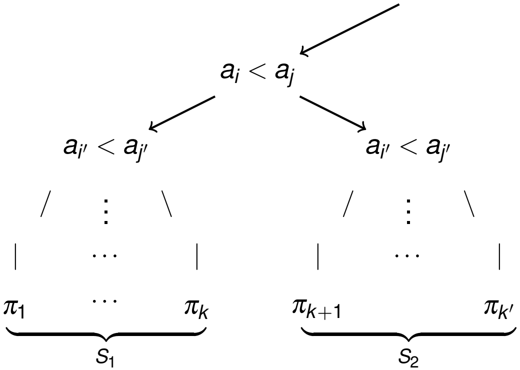

Either the set of permutations S_1 or S_2 is larger.
One must be at least half.

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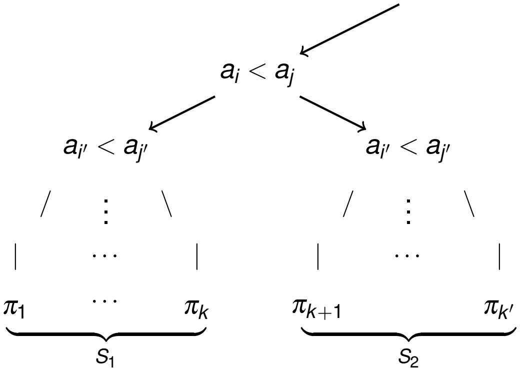

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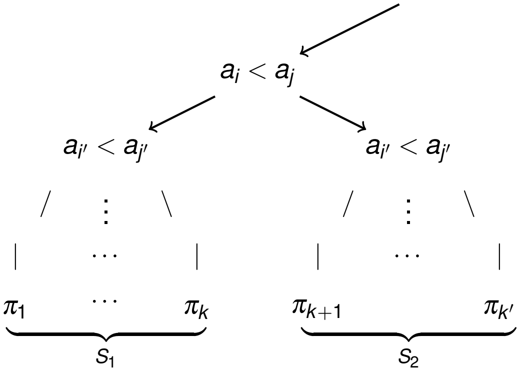

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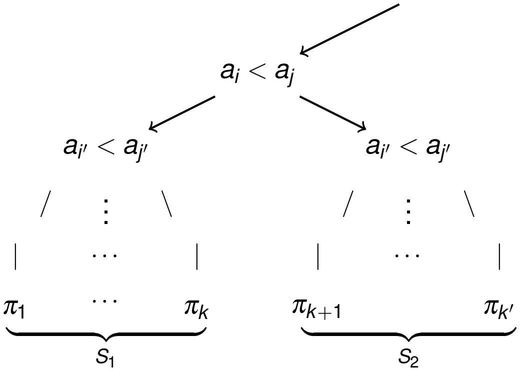

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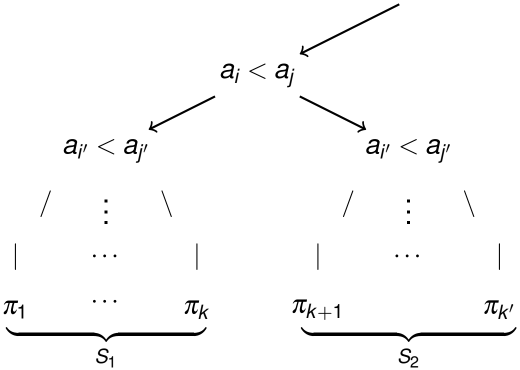

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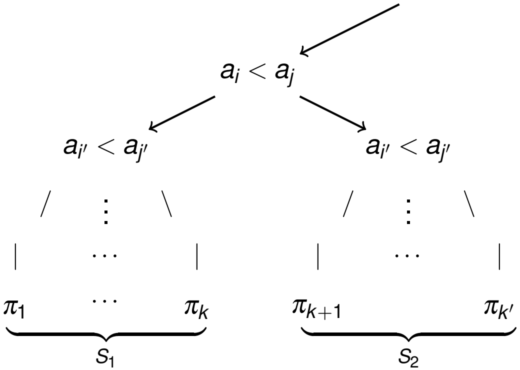

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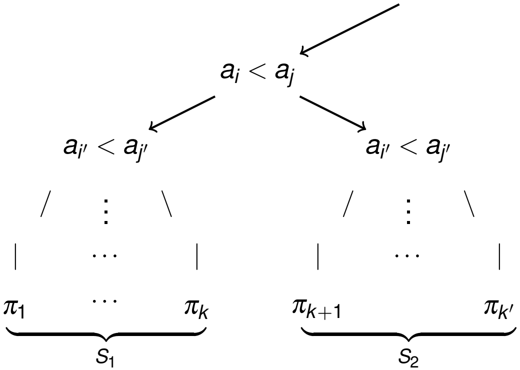

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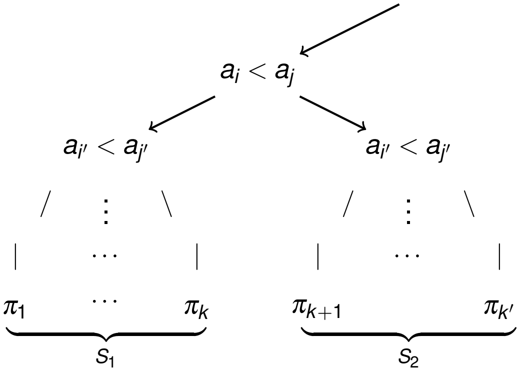

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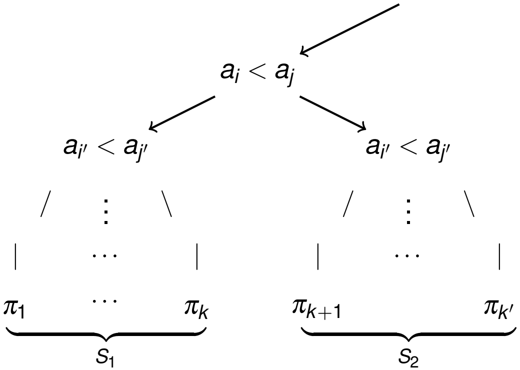

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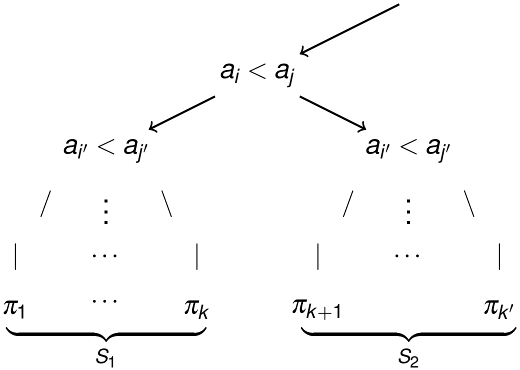

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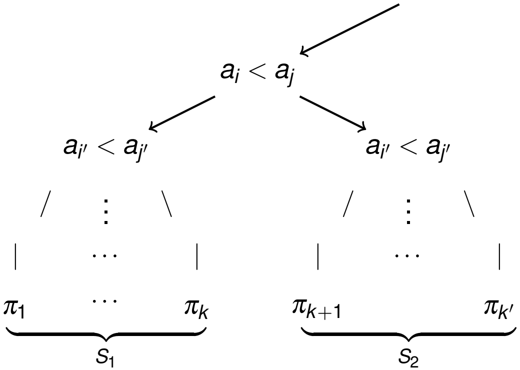

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(Recall from 61b: radix sort may be faster: $O(n)$.)

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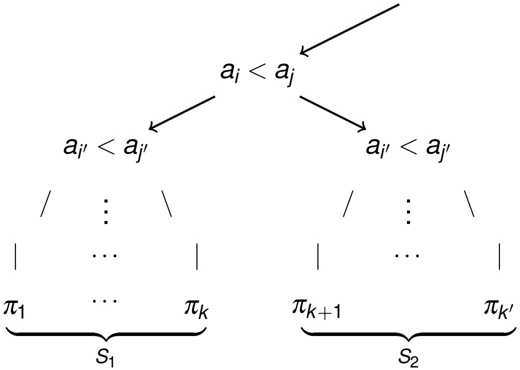

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Find the median element of a set of elements: $a_1, \ldots, a_n$.

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Better algorithm?

Solve a harder Problem: Selection.

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Output?

(A) 19

(B) 15

(C) 21

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else Select$(k - \vert S_L \vert - \vert S_v \vert, S_R)$	Select$(2, [48, 21, 17, 19])$

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Will eventually return 19, which is 7th element of list.

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Will eventually return 19, which is 7th element of list.

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Idea: Subroutine returns correct answer,

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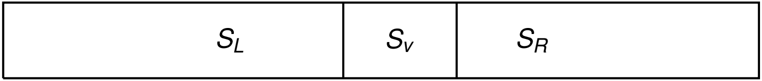

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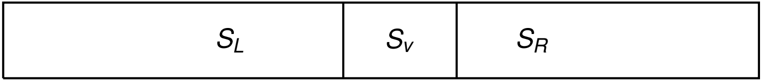

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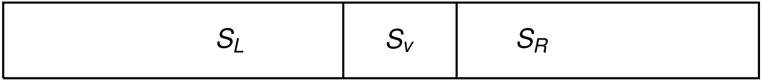

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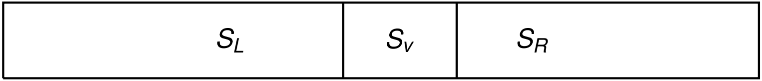

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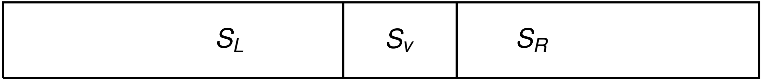

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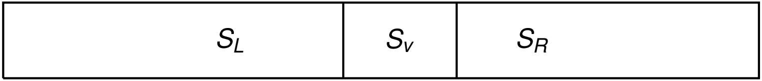

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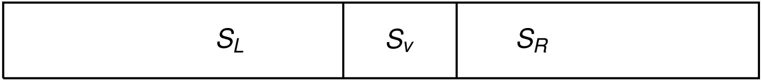

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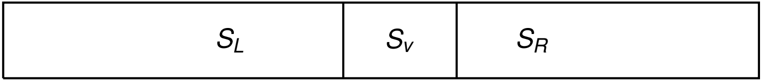

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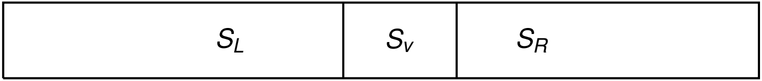

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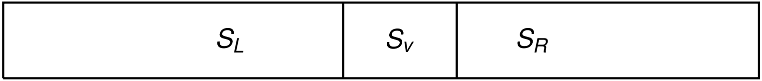

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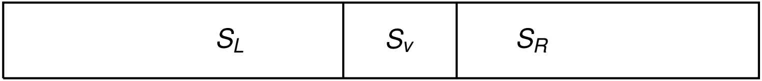

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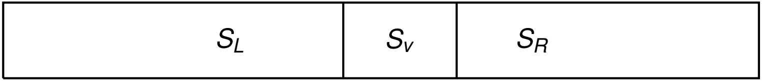

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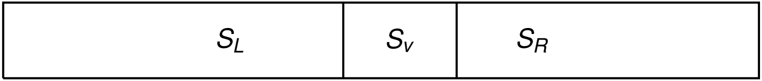

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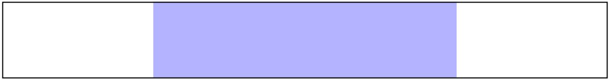

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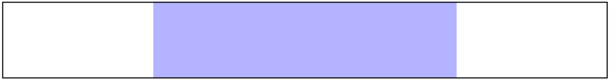

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Proof: $E[X] = 1 + \frac{1}{2}E[X]$
$\implies \frac{1}{2}E[X] = 1 \implies E[X] = 2.$ $\square$

Probability that random pivot elt in the middle half is $\geq \frac{1}{2}$.

Expected time to get a middle element is $E[X] \times O(n) = O(n)$.

Pick in the middle half subproblem size is $\leq \frac{3}{4}n$.

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Extra Example: Deterministic Selection.

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Choose rand. elt b from A.

Expected to be “in the middle”.

Instead: find elt that **must be** “in the middle.”

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$$T(n) \leq P(n) + T(\tfrac{7}{10}n) + O(n).$$

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$T(\frac{n}{5})$ for computing median of S.

$T(\frac{7}{10}n)$ for the recursive call in Select.

Bound Recurrence.

$$T(n) \leq T(\frac{n}{5}) + T(\frac{7}{10}n) + cn. \quad T(1) = c.$$

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Prove $T(n) \leq c'n$ for some c'.

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Prove $T(n) \leq c'n$ for some c'.

Induction Hypothesis: $T(n') \leq c'n'$ for $n' < n$.

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$$\begin{aligned}
T(n) &\leq T(\frac{n}{5}) + T(\frac{7}{10}n) + cn \\
&\leq c'\frac{n}{5} + c'\frac{7}{10}n + cn \\
&\leq c'\frac{9}{10}n + cn \\
&\leq c'n + (c - c'\frac{1}{10})n
\end{aligned}$$

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Choose $c' \ge 10c$

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Choose $c' \ge 10c \implies c - c'\frac{1}{10} < 0$

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Base Case: $c' \geq c$.

Bound Recurrence.

$T(n) \leq T(\frac{n}{5}) + T(\frac{7}{10}n) + cn. \quad T(1) = c.$

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$$\begin{aligned} T(n) &\leq T(\frac{n}{5}) + T(\frac{7}{10}n) + cn \\ &\leq c'\frac{n}{5} + c'\frac{7}{10}n + cn \\ &\leq c'\frac{9}{10}n + cn \\ &\leq c'n + (c - c'\frac{1}{10})n \end{aligned}$$

Choose $c' \geq 10c \implies c - c'\frac{1}{10} < 0 \implies T(n) \leq c'n.$

Base Case: $c' \geq c$. □

Bound Recurrence.

$$T(n) \le T(\tfrac{n}{5}) + T(\tfrac{7}{10}n) + cn. \quad T(1) = c.$$

Idea: $\frac{1}{5} + \frac{7}{10} < 1$. Problem sizes decrease geometrically.

Prove $T(n) \le c'n$ for some c'.

Induction Hypothesis: $T(n') \le c'n'$ for $n' < n$.

$$\begin{aligned} T(n) &\le T(\frac{n}{5}) + T(\frac{7}{10}n) + cn \\ &\le c'\frac{n}{5} + c'\frac{7}{10}n + cn \\ &\le c'\frac{9}{10}n + cn \\ &\le c'n + (c - c'\frac{1}{10})n \end{aligned}$$

Choose $c' \ge 10c \implies c - c'\frac{1}{10} < 0 \implies T(n) \le c'n$.

Base Case: $c' \ge c$. $\square$

Selection is $O(n)$ deterministic time!

Lecture in a minute.

MergeSort.

Lecture in a minute.

MergeSort.
- Sort two halves, put together.
  - Merge: two pointer scan.
  - $T(n) = 2T(\frac{n}{2}) + O(n)$.
  - $T(n) = O(n \log n)$.
- Also: iterative view.

Lecture in a minute.

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Sorting Lower Bound.

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$n!$ possible output orderings.

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Comparison splits outputs into 2.

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$\Omega(\log n!) = \Omega(n \log n)$ time.

Lecture in a minute.

MergeSort.
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Sorting Lower Bound.
 $n!$ possible output orderings.
 Comparison splits outputs into 2.
 $\Omega(\log n!) = \Omega(n\log n)$ time.

Median finding.
  Selection: more general, "strengthen induction."
  Random pivot element to split elements.
  Recurse on one subset.

Lecture in a minute.

MergeSort.

Sort two halves, put together.

Merge: two pointer scan.

$T(n) = 2T(\frac{n}{2}) + O(n)$.

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Expected Time Analysis:

$O(n)$ time to decrease size by $3/4$.

Lecture in a minute.

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Extra: Deterministic Pivot Selection.

Lecture in a minute.

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Sorting Lower Bound.

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