CS 170: Algorithms
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Lecture in a Minute

Depth First Search.
   Call explore until explore the whole graph.
Connected Components.
Tree/back edges.
   Back edge $\iff$ cycle
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Pre/Post Ordering.
  Interval of time “on stack”.
  Quick cycle test.
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Directed Graphs.
   Tree/Back/Forward/Cross edges.
   From pre/post!
   Back Edge $\iff$ cycle!
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Topological Sort.
   Alg 1: Inverse Post order number.
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Topological Sort.
- Alg 1: Inverse Post order number.
  - Inverse order of “stack” pop.
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Topological Sort.
  Alg 1: Inverse Post order number.
    Inverse order of “stack” pop.
  Alg 2: Peeling off sources.
Explore.

Graph $G = (V, E)$. 

Size of adjacency list representation?

Explore($v$):
2. For each edge $(v, w)$ in $E$
   3. if not visited[$w$]: Explore($w$)

Property:
All and only nodes reachable from $A$ are reached by explore.

Idea:
Reachable means there is a path, and there is no first unexplored node since the previous node would explore it.

Runtime:
$O(|V| + |E|)$.
The time is proportional to total size of the adjacency lists.
Explore.

Graph $G = (V, E)$.

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Size of adjacency list representation? $O(|V| + |E|)$. 

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Graph $G = (V, E)$.

Size of adjacency list representation? $O(|V| + |E|)$.

**Explore(v):**
2. For each edge (v,w) in E
3. if not visited[w]: Explore(w)
Explore.

Graph $G = (V, E)$.

Size of adjacency list representation? $O(|V| + |E|)$.

**Explore(v):**
1. Set $\text{visited}[v] := \text{true}$.
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Graph \( G = (V, E) \).

Size of adjacency list representation? \( O(|V| + |E|) \).

**Explore(v):**
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Size of adjacency list representation? $O(|V| + |E|)$.

**Explore**(v):
1. Set visited[v] := **true**.
2. For each edge (v,w) in E
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**Property:**
All and only nodes reachable from A are reached by explore.

**Idea:**
Reachable means there is a path, and there is no first unexplored node since the previous node would explore it.

**Runtime:** $O(|V| + |E|)$.
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Depth first search.

Process whole graph.
Depth first search.

Process whole graph.

**DFS(G)**

1: For each node $u$, 

Running time: $O(|V| + |E|)$. 

Intuitively: tree for each “connected component”. Several trees or Forest! Output connected components?
Depth first search.

Process whole graph.

**DFS(G)**
1: For each node \( u \),
2: \( \text{visited}[u] = \text{false} \).
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1: For each node \( u \),
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3: For each node \( u \),
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Depth first search.

Process whole graph.

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1: For each node $u$,
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Running time: $O(|V| + |E|)$. 
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Process whole graph.

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2: \hspace{1em} visited[$u$] = \textbf{false}.
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Several trees
Depth first search.

Process whole graph.

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Running time: $O(|V| + |E|)$.

Intuitively: tree for each “connected component”. Several trees or Forest! Output connected components?
DFS and connected components.

DFS(G):
0. Set cc := 0.
1. for each v in V:
2. if not visited[v]:
3. explore(v)
4. ccnum = ccnum+1

Each node will be labelled with connected component number.

Runtime: $O(|V| + |E|)$. 
DFS and connected components.

Change explore a bit:

explore(v):
2. previsit(v)
3. For each edge (v,w) in E
4. if not visited[w]: explore(w).
5. postvisit(v)

Previsit(v):

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**Connected Components.**

**explore(v):**
1. Set $\text{visited}[v] := \text{true}$.  
2. $\text{previsit}(v)$  
3. For each edge $(v,w)$ in $E$  
4. if not $\text{visited}[w]$: $\text{explore}(w)$.  
5. $\text{postvisit}(v)$

**Previsit(v):**
1. Set $\text{cc}[v] := \text{ccnum}$.  

**DFS(G):**  
0. Set $\text{cc} := 0$.  
1. for each $v$ in $V$:  
   2. if not $\text{visited}[v]$:  
      3. $\text{explore}(v)$  
      4. $\text{ccnum} = \text{ccnum}+1$
Connected Components.

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**Connected Components.**

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Introspection: pre/post.

Previsit(v):
2. clock := clock+1

Postvisit(v):
2. clock := clock+1

DFS(G):
0. Set clock := 0.
···

Clock: goes up to 2 times number of tree edges.

First pre:

Property:
For any two nodes, u and v,
[pre(u), post(u)] and [pre(v), post(v)] are either disjoint or one is contained in other.
Interval is "clock interval on stack."

u ... v ...
Either both on stack at some point (contained) or not (disjoint.)

Let's just watch it work!
Introspection: pre/post.

**Previsit(v):**
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**Previsit(v):**
2. clock := clock + 1

**DFS(G):**
0. Set clock := 0.
   ...

**Postvisit(v):**
2. clock := clock + 1
Introspection: pre/post.

Previsit(v):
2. clock := clock+1

Postvisit(v):
2. clock := clock+1

DFS(G):
0. Set clock := 0.
   ...

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0. Set clock := 0.
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Previsit(v):
2. clock := clock + 1

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DFS(G):
0. Set clock := 0.

... 

Clock: goes up to 2 times number of tree edges.
First pre:
Introspection: pre/post.

Previsit\(v\):
1. Set \(\text{pre}[v] := \text{clock}\).
2. \(\text{clock} := \text{clock}+1\)

Postvisit\(v\):
1. Set \(\text{post}[v] := \text{clock}\).
2. \(\text{clock} := \text{clock}+1\)

DFS\((G)\):
0. Set \(\text{clock} := 0\).

...  

Clock: goes up to 2 times number of tree edges.
First pre: 0
Introspection: pre/post.

**Previsit(v):**
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**DFS(G):**
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```
  u
  ...
  v
  ...
```

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[pre(v),post(v)] are either disjoint or one is contained in other.

\[
\begin{array}{c}
\text{u} \\
\cdots \\
\text{v} \\
\cdots \\
\end{array}
\]

Interval is “clock interval on stack.”
Either both on stack at some point (contained) or not (disjoint.)
Let’s just watch it work!
Example: Pre/Post numbering.

Explored edge \((u, v)\) first from \(u\).

Tree edge iff \([\text{pre}[v], \text{post}[v]]\) \(\in [\text{pre}[u], \text{post}[u]]\). \(u\) on stack before \(v\).

Back edge iff \([\text{pre}[u], \text{post}[u]]\) \(\in [\text{pre}[v], \text{post}[v]]\). \(v\) on stack when \(v\) on stack. Path from \(v\) to \(u\) \(!=\) Cycle! No edge between \(u\) and \(v\) if disjoint intervals.
Example: Pre/Post numbering.
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Back edge iff \([\text{pre}([u]), \text{post}([u])] \in [\text{pre}([v]), \text{post}([v])]\).

\(v\) on stack when \(v\) on stack. Path from \(v\) to \(u\) is a cycle!

No edge between \(u\) and \(v\) if disjoint intervals.
Example: Pre/Post numbering.

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Tree edge iff \([\text{pre}\[v\]], \text{post}\[v\]\] \(\in\) \([\text{pre}\[u\]], \text{post}\[u\]\]. \(u\) on stack before \(v\).

Back edge iff \([\text{pre}\[u\]], \text{post}\[u\]\] \(\in\) \([\text{pre}\[v\]], \text{post}\[v\]\]. \(v\) on stack when \(v\) on stack. Path from \(v\) to \(u\)! Cycle!

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\(v\) on stack when \(v\) on stack.

path from \(v\) to \(u\) ! cycle!

no edge between \(u\) and \(v\) if disjoint intervals.
Example: Pre/Post numbering.

Explored edge \((u, v)\) first from \(u\).

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Path from \(v\) to \(u\) is a cycle!

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Example: Pre/Post numbering.

Explored edge $(u, v)$ first from $u$.

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Tree edge iff \([\text{pre}_v, \text{post}_v] \in [\text{pre}_u, \text{post}_u]\).

\(u\) on stack before \(v\).

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Directed graphs.

\[ G = (V, E) \]
Directed graphs.

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vertices \( V \).
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Edge: \((u, v)\)
From \( u \) to \( v \).
Directed graphs.

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Edge: \( (u, v) \)
From \( u \) to \( v \).
\( \text{Tail} - u \)
Directed graphs.

\[ G = (V, E) \]

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Edge: \( (u, v) \)

From \( u \) to \( v \).

Tail – \( u \)

Head – \( v \)
Directed graphs.

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vertices \( V \).
edges \( E \subseteq V \times V \).

Edge: \( (u, v) \)
From \( u \) to \( v \).
Tail – \( u \)
Head – \( v \)
DFS on directed graphs.

Terminology:

- **Root**: Starting point.
- **v is ancestor of u**: v on path from/to root.
- **v is descendant of u**: u is an ancestor of v.
DFS on directed graphs.

**Terminology:**

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\( v \) is ancestor of \( u \):

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DFS on directed graphs.

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*Root*: Starting point.

$v$ *is ancestor of* $u$:
- $v$ on path from/to root.

$v$ *is descendant of* $u$:
- $u$ is an ancestor of $v$. 
DFS on directed graphs.

Terminology:

Root: Starting point.

$v$ is ancestor of $u$:
$v$ on path from/to root.

$v$ is descendant of $u$:
$u$ is an ancestor of $v$. 
Depth first search: directed.

Tree/forward edge \((u, v)\): int\((v)\) in int\((u)\).

Forward \((A, F)\): \([10,11]\) in \([0,13]\) or \([0, [10,11], 13]\)

Back edge \((u, v)\): int\((v)\) contains int\((u)\).

\((C, B)\): \([3,4]\) in \([1,8]\) or \([1, [3, 4], 8]\)

Cross edge \((u, v)\): int\((v)\) before int\((u)\).

\((F, D)\): \([2,5]\) before \([10,11]\)
Depth first search: directed.

- **Tree/forward edge** \((u, v)\): int\((v)\) in int\((u)\).
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**Tree/forward edge** \((u, v)\): \(v \in \text{int}(u)\).

**Forward** \((A, F)\): \([10, 11] \in [0, 13] \cup [0, [10, 11], 13]\)

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Depth first search: directed.

Tree/forward edge \((u, v)\): \(v \in u\).

Forward \((A, F)\): \([10, 11] \in [0, 13] \) or \([0, [10, 11], 13]\) 

Back edge \((u, v)\): \(v \) contains \(u\).

\((C, B)\): \([3, 4] \in [1, 8] \) or \([1, [3, 4], 8]\) 

Cross edge \((u, v)\): \(v \) before \(u\).

\((F, D)\): \([2, 5]\) before \([10, 11]\)
Depth first search: directed.

A directed tree/forward edge \((u, v)\): \(\text{int}(v) \in \text{int}(u)\).

A forward edge \((A, F)\): \([10, 11]\) in \([0, 13]\) or \([0, [10, 11], 13]\)

A back edge \((u, v)\): \(\text{int}(v)\) contains \(\text{int}(u)\). \((C, B)\): \([3, 4]\) in \([1, 8]\) or \([1, [3, 4], 8]\)

A cross edge \((u, v)\): \(\text{int}(v)\) before \(\text{int}(u)\). \((F, D)\): \([2, 5]\) before \([10, 11]\)
Depth first search: directed.

Tree/forward edge \((u, v)\): int\((v)\) in int\((u)\).

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Tree/forward edge \((u, v)\): \(\text{int}(v)\) in \(\text{int}(u)\).
Depth first search: directed.

**Tree/forward edge** $(u, v)$: $\text{int}(v)$ in $\text{int}(u)$.

Forward $(A, F)$: $[10,11]$ in $[0,13]$ or $[0,[10,11],13]$
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Edge: \((u, v)\)
Directed Acyclic Graphs: Depth First Search

Edge: \((u, v)\)
From \(u\) to \(v\).
Directed Acyclic Graphs: Depth First Search

Edge: \((u, v)\)
From \(u\) to \(v\).
    Tail – \(u\)
Directed Acyclic Graphs: Depth First Search

Edge: \((u, v)\)
From \(u\) to \(v\).
Tail – \(u\)
Head – \(v\)
Directed Acyclic Graphs: Depth First Search

Edge: \((u, v)\)
   From \(u\) to \(v\).
   Tail – \(u\)
   Head – \(v\)

Tree edge – “Direct call tree of explore.”
Directed Acyclic Graphs: Depth First Search

Edge: \((u, v)\)
- From \(u\) to \(v\).
  - Tail – \(u\)
  - Head – \(v\)

Tree edge – “Direct call tree of explore.”

Forward edge – “Edge to descendant (not in tree)”
Directed Acyclic Graphs: Depth First Search

Edge: \((u, v)\)
   From \(u\) to \(v\).
   Tail – \(u\)
   Head – \(v\)

Tree edge – “Direct call tree of explore.”
Forward edge – “Edge to descendant (not in tree)”
Back edge – “Edge to ancestor”
Directed Acyclic Graphs: Depth First Search

Edge: \((u, v)\)
From \(u\) to \(v\).
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Cross edge – None of the above.
  \(v\) already explored before \(u\) is visited.
Directed Acyclic Graphs: Depth First Search

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   From \(u\) to \(v\).
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Directed Acyclic Graph

Directed Graph...
Directed Acyclic Graph

Directed Graph ...without cycles.
Directed Acyclic Graph

Directed Graph ...without cycles. Cycle: $v_0 \rightarrow v_1 \rightarrow \ldots v_k \rightarrow v_0$. 
Directed Acyclic Graph

Directed Graph ...without cycles. Cycle: $v_0 \rightarrow v_1 \rightarrow \ldots v_k \rightarrow v_0$. Why?
Directed Acyclic Graph

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Hello

Goodbye
Directed Acyclic Graph

Directed Graph ...without cycles. Cycle: \( v_0 \rightarrow v_1 \rightarrow \ldots \rightarrow v_k \rightarrow v_0 \). Why?

“Hello” before “Goodbye”
Directed Acyclic Graph

Directed Graph ...without cycles. Cycle: $v_0 \rightarrow v_1 \rightarrow \ldots v_k \rightarrow v_0$. Why?

“Hello” before “Goodbye”
Example.
Example.

Acyclic Graph?
Example.

Acyclic Graph?
Acyclic Graph?
Example.

Acyclic Graph?
Example.

Acyclic Graph?
Example.

Acyclic Graph?
Depth first search: directed.

Back edge $(u, v)$: int$(v)$ contains int$(u)$. 

int$(C) = [3, 4]$ and int$(B) = [1, 8]$.

Back edge $(u, v)$ means cycle! ⇒ not acyclic!
Depth first search: directed.

Back edge \((u, v)\): int\((v)\) contains int\((u)\).

\[\text{int}(C) = [3, 4]\]
\[\text{int}(B) = [1, 8]\]

Back edge \((u, v)\): edge to ancestor tree edges from \(v\) to \(u\).

Back edge means cycle! \(\Rightarrow\) not acyclic!
Depth first search: directed.

Back edge \((u, v)\): \(\text{int}(v)\) contains \(\text{int}(u)\).
Depth first search: directed.

Back edge \((u, v)\): \(\text{int}(v)\) contains \(\text{int}(u)\).

\(\text{int}(C) = [3, 4]\) and \(\text{int}(B) = [1, 8]\).
Depth first search: directed.

Back edge $(u, v)$: $\text{int}(v)$ contains $\text{int}(u)$. 

$\text{int}(C) = [3, 4]$ and $\text{int}(B) = [1, 8]$. 

Back edge $(u, v)$
Depth first search: directed.

Back edge \((u, v)\): \(\text{int}(v)\) contains \(\text{int}(u)\).

\(\text{int}(C) = [3, 4]\) and \(\text{int}(B) = [1, 8]\).

Back edge \((u, v)\)
....edge to ancestor
Depth first search: directed.

Back edge \((u, v)\): \(\text{int}(v)\) contains \(\text{int}(u)\).

\(\text{int}(C) = [3, 4]\) and \(\text{int}(B) = [1, 8]\).

Back edge \((u, v)\)
edge to ancestor
tree edges from \(v\) to \(u\).
Depth first search: directed.

Back edge \((u, v)\): \(\text{int}(v)\) contains \(\text{int}(u)\).

\(\text{int}(C) = [3, 4]\) and \(\text{int}(B) = [1, 8]\).

Back edge means cycle!
Depth first search: directed.

Back edge \((u, v)\): \(\text{int}(v)\) contains \(\text{int}(u)\).

\(\text{int}(C) = [3, 4]\) and \(\text{int}(B) = [1, 8]\).

Back edge means cycle! \(\Rightarrow\) not acyclic!
Testing for cycle.

**Thm:** A graph has a cycle if and only if there is back edge.
Testing for cycle.

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**Proof:**
Testing for cycle.

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**Proof:**
Back edge $\implies$ cycle!
Testing for cycle.

**Thm:** A graph has a cycle if and only if there is back edge.

**Proof:**
Back edge $\Rightarrow$ cycle!

There is a cycle
Testing for cycle.

**Thm:** A graph has a cycle if and only if there is back edge.

**Proof:**
Back edge $\implies$ cycle!

There is a cycle

$V_0 \rightarrow V_1$
Testing for cycle.

**Thm:** A graph has a cycle if and only if there is back edge.

**Proof:**
Back edge $\iff$ cycle!

There is a cycle

$v_0 \rightarrow v_1 \rightarrow v_2$
Thm: A graph has a cycle if and only if there is back edge.

Proof:
Back edge $\implies$ cycle!

There is a cycle

$\nu_0 \rightarrow \nu_1 \rightarrow \nu_2 \cdots \rightarrow \nu_k \rightarrow \nu_0$
Testing for cycle.

**Thm:** A graph has a cycle if and only if there is back edge.

**Proof:**
Back edge $\implies$ cycle!

There is a cycle

$v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k \rightarrow v_0$

Assume that $v_0$ is the first node explored.
   (without loss of generality since can renumber vertices.)
Testing for cycle.

**Thm:** A graph has a cycle if and only if there is back edge.

**Proof:**

Back edge $\implies$ cycle!

There is a cycle

$v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k \rightarrow v_0$

Assume that $v_0$ is the first node explored.

(without loss of generality since can renumber vertices.)

All nodes on cycle explored when \textit{explore}(v_0) returns
Thm: A graph has a cycle if and only if there is back edge.

Proof:
Back edge $\implies$ cycle!

There is a cycle

$v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k \rightarrow v_0$

Assume that $v_0$ is the first node explored.
(without loss of generality since can renumber vertices.)

All nodes on cycle explored when `explore($v_0$) returns

For each $v_i$: int[$v_i$] $\in$ int[$v_0$]!
Thm: A graph has a cycle if and only if there is a back edge.

Proof:
Back edge $\iff$ cycle!

There is a cycle

$v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k \rightarrow v_0$

Assume that $v_0$ is the first node explored.

(without loss of generality since can renumber vertices.)

All nodes on cycle explored when explore($v_0$) returns

For each $v_i$: int[$v_i$] $\in$ int[$v_0$]!

$\implies (v_k, v_0)$ is a back edge.
Thm: A graph has a cycle if and only if there is back edge.

Proof:
Back edge $\implies$ cycle!

There is a cycle

$v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k \rightarrow v_0$

Assume that $v_0$ is the first node explored.
(without loss of generality since can renumber vertices.)

All nodes on cycle explored when $\text{explore}(v_0)$ returns

For each $v_i$: int[$v_i$] $\in$ int[$v_0$]!

$\implies$ $(v_k, v_0)$ is a back edge.

Cycle $\implies$ back edge!
Testing for cycle.

**Thm:** A graph has a cycle if and only if there is back edge.

**Proof:**
Back edge $\implies$ cycle!

There is a cycle

\[ v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k \rightarrow v_0 \]

Assume that $v_0$ is the first node explored.

(Without loss of generality since can renumber vertices.)

All nodes on cycle explored when `explore`(\(v_0\)) returns

For each \(v_i: \text{int}[v_i] \in \text{int}[v_0]!\)

$\implies (v_k, v_0)$ is a back edge.

Cycle $\implies$ back edge!
Thm: A graph has a cycle if and only if there is back edge.
Fast checking algorithm.

**Thm:** A graph has a cycle if and only if there is a back edge.
Run DFS.
Fast checking algorithm.

**Thm:** A graph has a cycle if and only if there is a back edge.

Run DFS. $O(|V| + |E|)$ time.
Thm: A graph has a cycle if and only if there is back edge.
Run DFS. $O(|V| + |E|)$ time.
For each edge $(u, v)$: is $\text{int}(u)$ in $\text{int}(v)$. 
**Thm:** A graph has a cycle if and only if there is back edge.

Run DFS. $O(|V| + |E|)$ time.

For each edge $(u, v)$: is int$(u)$ in int$(v)$. $O(|E|)$ time.
Fast checking algorithm.

**Thm:** A graph has a cycle if and only if there is back edge.

Run DFS. $O(|V| + |E|)$ time.
For each edge $(u, v)$: is int$(u)$ in int$(v)$. $O(|E|)$ time.

$O(|V| + |E|)$ time algorithm for checking if graph is acyclic.
Directed Acyclic Graph

Hello

Goodbye

“Hello” before “Goodbye”
Directed Acyclic Graph

“Hello” before “Goodbye”

No cycles!
Directed Acyclic Graph

“Hello” before “Goodbye”

No cycles! Can tell in linear time!
Directed Acyclic Graph

“Hello” before “Goodbye”

No cycles! Can tell in linear time!

Ohhh...
Directed Acyclic Graph

“Hello” before “Goodbye”

No cycles! Can tell in linear time!

Ohhh...Kayyyy...
Directed Acyclic Graph

```
Hello
```

```
Goodbye
```

“Hello” before “Goodbye”

No cycles! Can tell in linear time!

Ohhh...Kayyyy...

Really want to find ordering for build!
Directed Acyclic Graph

“Hello” before “Goodbye”

No cycles! Can tell in linear time!

Ohhh...Kayyyy...
Really want to find ordering for build!
Where things are cool!
**Topological Sort:** For $G = (V, E)$, find ordering where each edge goes from earlier vertex to later in acyclic graph.
**Topological Sort:** For $G = (V, E)$, find ordering where each edge goes from earlier vertex to later in acyclic graph.
Linearize.

**Topological Sort**: For $G = (V, E)$, find ordering where each edge goes from earlier vertex to later in acyclic graph.
Topological Sort Example.

A linear order: A, F, E, B, G, D, C.

In DFS: When is A popped off stack? Last!

E second to last.
Topological Sort Example.

A linear order:
Topological Sort Example.

A linear order:

\[ A, F, E, B, G, D, C? \]
Topological Sort Example.

A linear order:

A linear order:

\[ A, F, E, B, G, D, C? \] Nope.

\[ A, E, F, B, G, D, C \]
Topological Sort Example.

A linear order:

A, E, F, B, G, D, C

In DFS: When is A popped off stack?
Topological Sort Example.

A linear order:

A, E, F, B, G, D, C

In DFS: When is A popped off stack?
Last!
Topological Sort Example.

A linear order:

\[ A, F, E, B, G, D, C? \]  Nope.

\[ A, E, F, B, G, D, C \]

In DFS: When is \( A \) popped off stack?

Last! \( E \)
Topological Sort Example.

A linear order:

\[ A, F, E, B, G, D, C? \text{ Nope.} \]

\[ A, E, F, B, G, D, C \]

In DFS: When is \( A \) popped off stack?

Last! \( E \) second to last.
Topological Sort Example.

A linear order:

\[ A, F, E, B, G, D, C? \text{ Nope.} \]
\[ A, E, F, B, G, D, C \]

In DFS: When is \( A \) popped off stack?

Last! \( E \) second to last. ...
Property: Every edge in a DAG \((u, v)\) has \(post(u) > post(v)\).
No back edges!
**Property:** Every edge in a DAG \((u, v)\) has \(post(u) > post(v)\).

No back edges!

Tree and Forward edge \((u, v)\):

\(\text{int}(u)\) contains \(\text{int}(v)\):

\(\text{pre}(u), \text{pre}[v], \text{post}[v], \text{post}[u]\)
Topological Sort: DFS

**Property:** Every edge in a DAG \((u, v)\) has \(post(u) > post(v)\).
No back edges!
Tree and Forward edge \((u, v)\):
\[\text{int}(u) \text{ contains } \text{int}(v): pre(u), pre[v], post[v], post[u]\]
Cross edge \((u, v)\): \text{int}(u) > \text{int}(v)\)
Topological Sort: DFS

Property: Every edge in a DAG \((u, v)\) has \(post(u) > post(v)\). No back edges!
Tree and Forward edge \((u, v)\):
  int\((u)\) contains int\((v)\): \(pre(u), pre[v], post[v], post[u]\)
Cross edge \((u, v)\): int\((u)\) > int\((v)\)
Top Sort: output in reverse post order number.
Property: Every edge in a DAG \((u, v)\) has \(post(u) \geq post(v)\). No back edges!

Tree and Forward edge \((u, v)\):
- \(int(u)\) contains \(int(v)\): \(pre(u), pre[v], post[v], post[u]\)

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Top Sort: output in reverse post order number.

Runtime: \(O(|V| + |E|)\).
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..procedure PostVisit outputs during DFS
**Topological Sort: DFS**

**Property:** Every edge in a DAG \((u, v)\) has \(post(u) > post(v)\).

No back edges!

Tree and Forward edge \((u, v)\):

- \(\text{int}(u)\) contains \(\text{int}(v)\): \(\text{pre}(u), \text{pre}[v], \text{post}[v], \text{post}[u]\)

Cross edge \((u, v)\): \(\text{int}(u) > \text{int}(v)\)

Top Sort: output in reverse post order number.

Runtime: \(O(|V| + |E|)\).

..procedure PostVisit outputs during DFS

..reverse.
Source/sinks in a DAG.

Source is node with no incoming arcs.
Source/sinks in a DAG.

**Source** is node with no incoming arcs.
**Sink** is node with no outcoming arcs.
Source/sinks in a DAG.

**Source** is node with no incoming arcs.
**Sink** is node with no outcoming arcs.

Highest post order node is source.
**Source** is node with no incoming arcs.  
**Sink** is node with no outcoming arcs.

Highest post order node is source.  
Lowest post order node is sink.
Source/sinks in a DAG.

**Source** is node with no incoming arcs.  
**Sink** is node with no outcoming arcs.  

Highest post order node is source.  
Lowest post order node is sink.  

**Property:** Every DAG has at least one source and sink.
**Source** is node with no incoming arcs.
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Highest post order node is source.
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**Property:** Every DAG has at least one source and sink.

Topological Sort Algorithm: Find source, output, repeat.
Source/sinks in a DAG.

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**Property:** Every DAG has at least one source and sink.

Topological Sort Algorithm: Find source, output, repeat.
Naively: $O(nm)$.
Source and sinks in a DAG.

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Naively: $O(nm)$.. there is a better implementation.
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Topological Sort Algorithm: Find source, output, repeat.

Naively: $O(nm)$.. there is a better implementation.
Useful on Monday.
Lecture in a Minute

Depth First Search.
  Call explore until explore the whole graph.
Connected Components.
  Tree/back edges.
  Back edge $\iff$ cycle

Directed Graphs.
  Tree/Back/Forward/Cross edges.
  From pre/post!
  Back Edge $\iff$ cycle!

Topological Sort.
  Alg 1: Inverse Post order number.
  Inverse order of "stack" pop.
  Alg 2: Peeling off sources.
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   Call explore until explore the whole graph.
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Pre/Post Ordering.
   Interval of time “on stack”.
Quick cycle test.
Lecture in a Minute

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