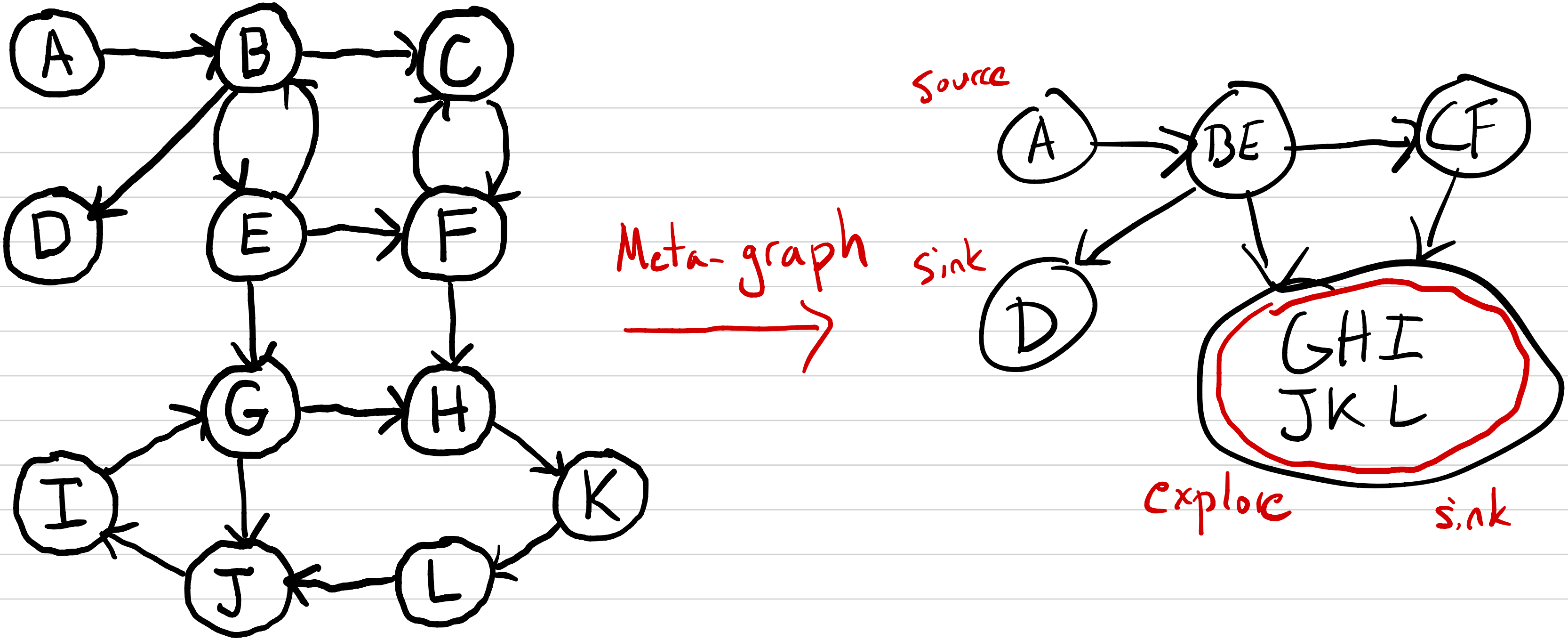
Strongly
Connected
Components

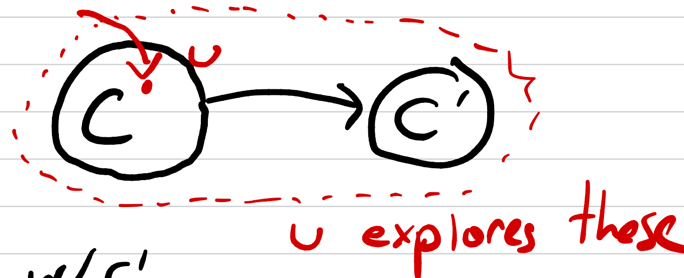


Magic algorithm: gives us vertex u in sink SCC via DFS! (with a twist...)

Suppose we run DFS.
For all SCCs C, define finish(C) = C's highest post(v)

Claim: Let $C \to C'$ be SCC's.
Then finish(C) > finish(C').

Pf: (i) Suppose DFS visits C first.
Then post(v) > finish(C').



(ii) Suppose DFS visits C' first.
Then only visits C after done w/ C'.

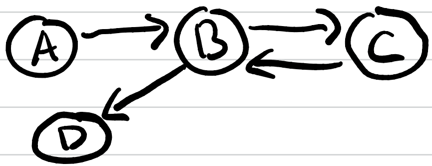
∴ all posts in C
> finish(C').

Can't happen!
Graph is DAG.

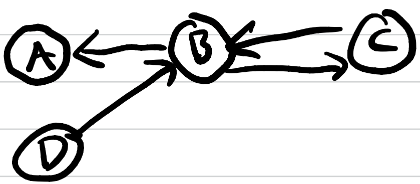
Claim: The highest post(v) is in source SCC.
Assume not.

The reverse graph

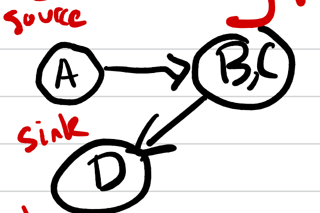
G

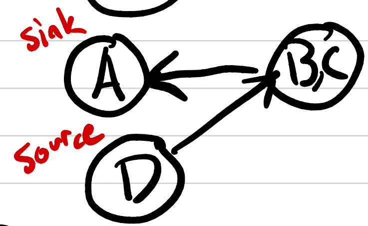


G^R



Meta graph





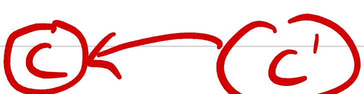
Claim: G and G^R have same SCCs.

In meta graphs:
- edges are reversed
- sources and sinks are swapped

Run DFS on G^R. Compute $post_R$ values.

u w/ highest $post_R$:
- (in G^R) in source SCC
- (in G) in sink SCC

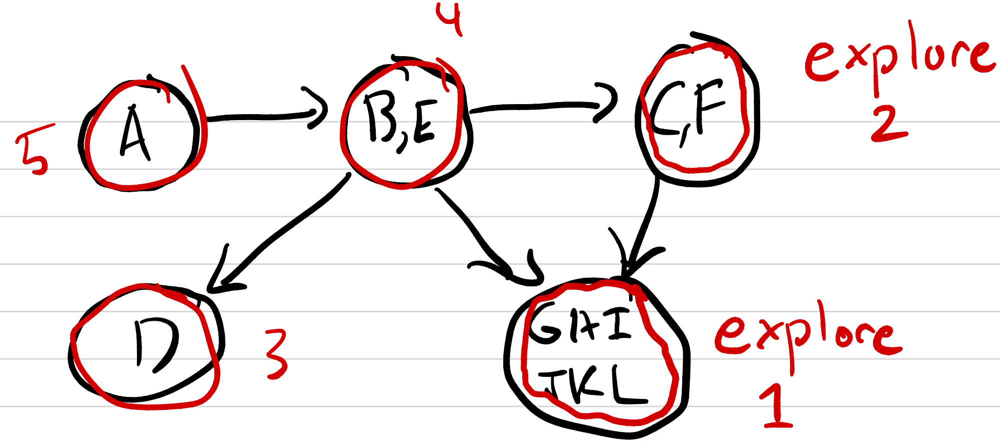
If  (in G^R) then highest $post_R$ in C > in C'

 (in G)

SCC algorithm

$C \leftarrow C'$

highest $post_R(u)$ in C > in C'




```
explore(G, u)
    visited[u] = true
    sccnum[u] = count

    for v s.t. (u,v) ∈ E
        if visited[v] = false
            explore(G,v)
```

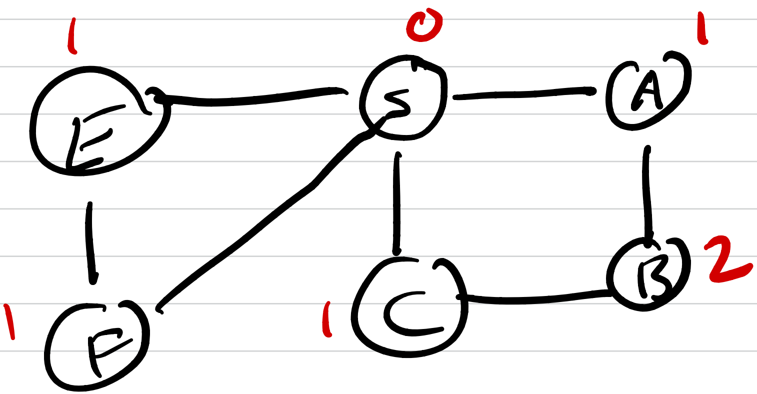
```
FindSCCs(G)
    for all u, visited[u] = false

    Run DFS on G^R
        to compute post_R

    Count = 1
    for u ∈ V (in reverse post_R order)
        if visited[u] = false
            explore(G,u)
            count = count + 1
```

Paths in Graphs

Single-source shortest paths (SSSP)

Input: Graph G, "source" vertex $s \in V$

Output: $\forall\, v \in V$, $d(s,v)$ = length of shortest path from s to v



Unweighted: all edges length 1

Breadth-first search

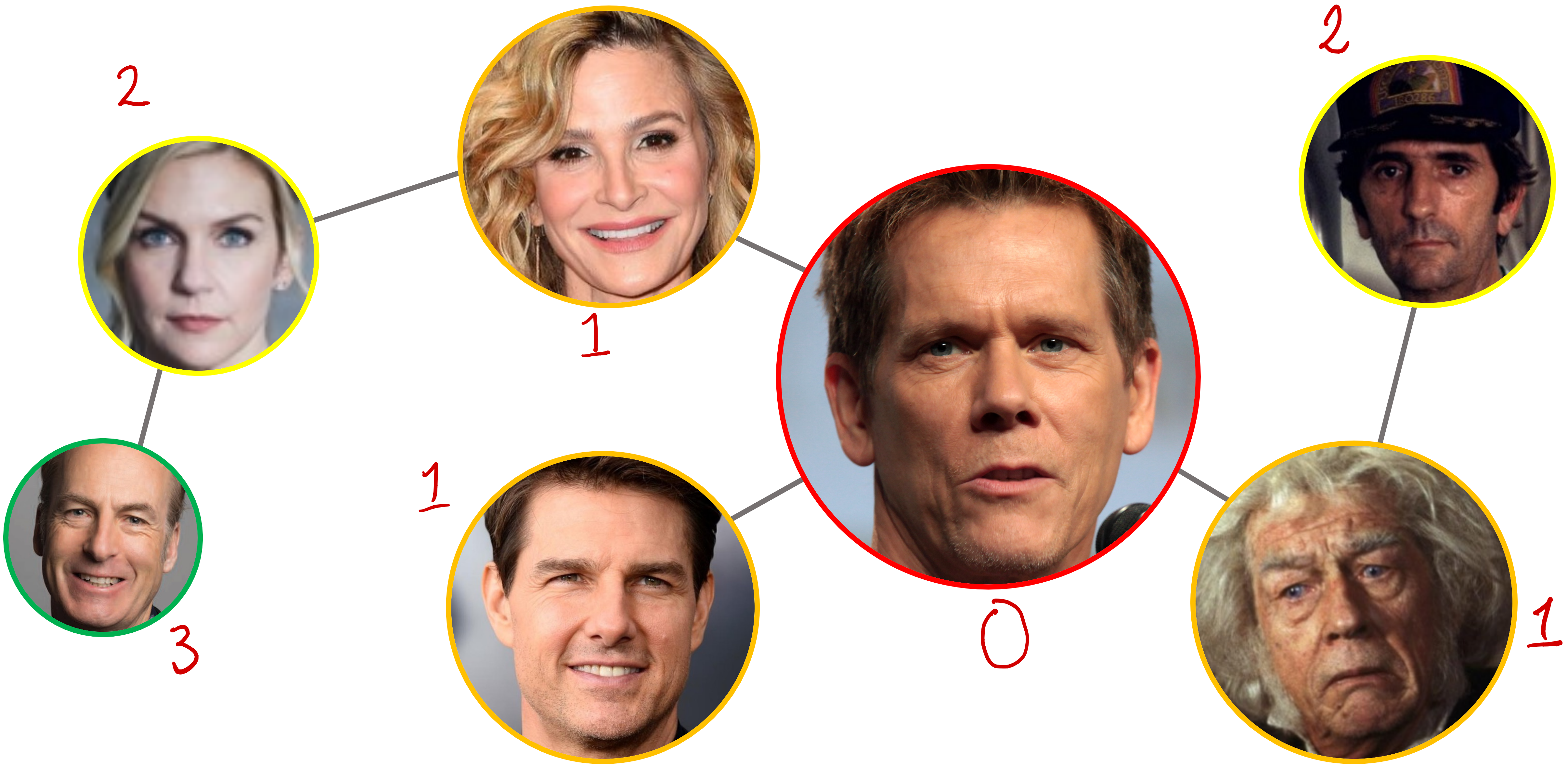
Positive lengths: $\ell : E \to \{1, 2, 3, \dots\}$

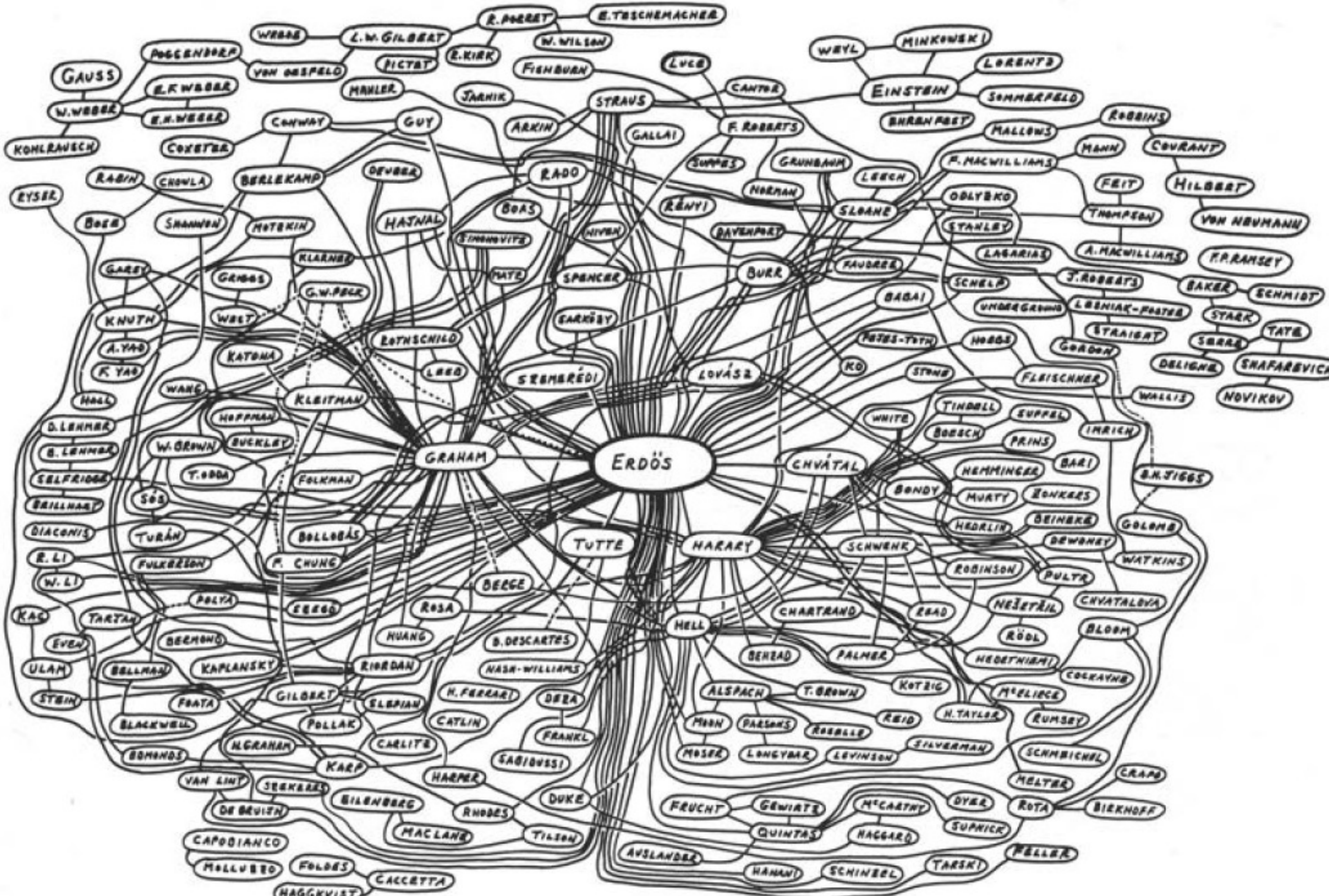
Dijkstra

Arbitrary length edges

Bellman-Ford

Application: Kevin Bacon number

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E. TESCHEMACHER
L.W. GILBERT
W. WILSON
E. KIRK
PICTET
POGGENDORF
GAUSS
E.F. WEBER
W. WEBER
E.H. WEBER
KOHLRAUSCH
FISHBURN
LUCE
MAHLER
JARNIK
STRAUS
CANTOR
WEYL
MINKOWSKI
LORENTZ
EINSTEIN
SOMMERFELD
EHRENFEST
CONWAY
GUY
COXETER
ARKIN
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COURANT
RABIN
CHOWLA
BERLEKAMP
DEUBER
RADO
SUPPES
GRUNBAUM
LEECH
F. MACWILLIAMS
FEIT
HILBERT
RYSER
BOSE
SHANNON
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BRILLHART
SÓS
HEDRLIN
BEINEKE
GOLOMB
DIACONIS
TURÁN
BOLLOBÁS
TUTTE
HARARY
SCHWENK
R. LI
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VAN LINT
SZEKERES
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DUKE
DYER
FRUCHT
GEWIRTZ
McCARTHY
ROTA
BIRKHOFF
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RHODES
QUINTAS
SUPNICK
HAGGARD
MAC LANE
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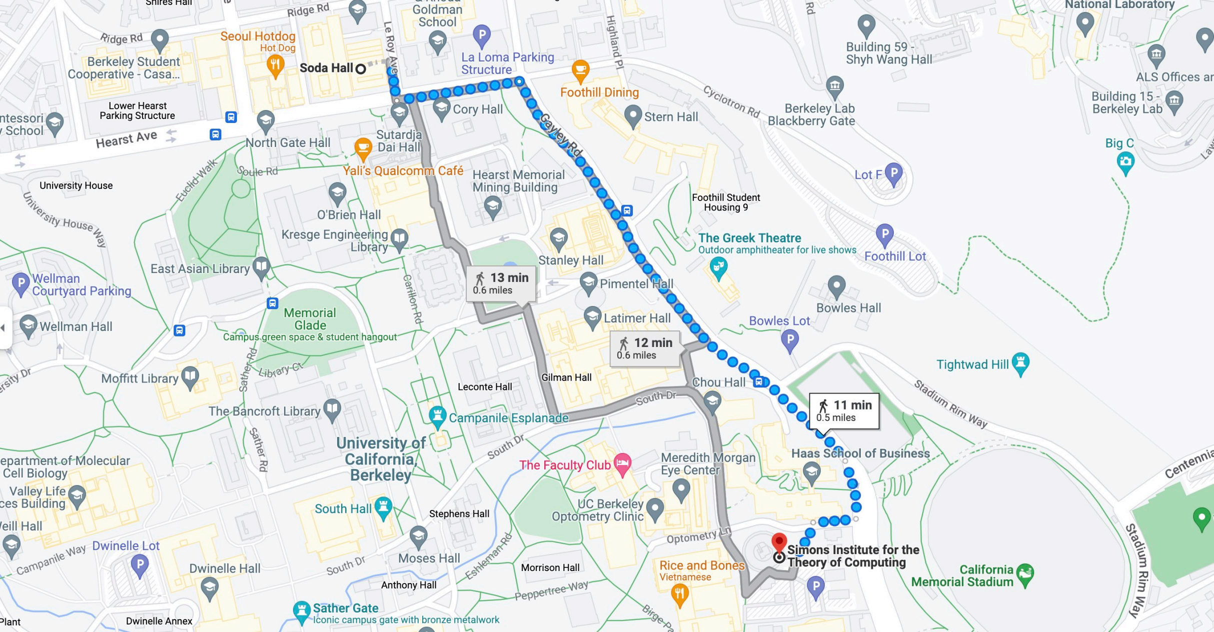
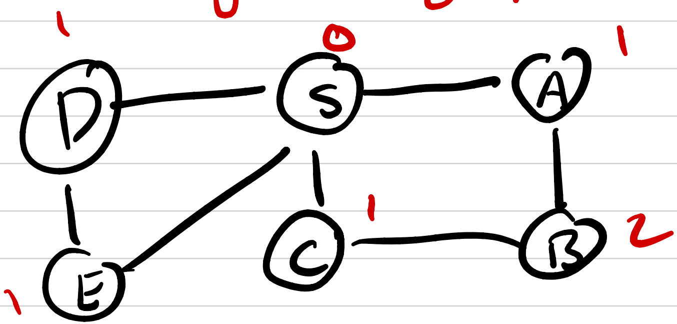
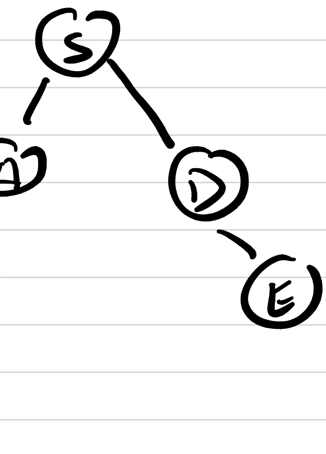
CACCETTA
HAGGKVIST

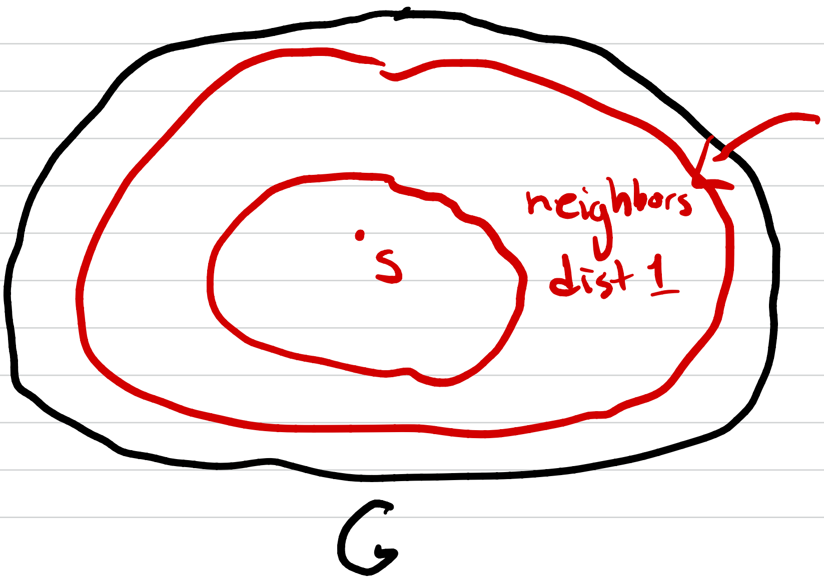

Soda Hall
Simons Institute for the Theory of Computing
13 min
0.6 miles
12 min
0.6 miles
11 min
0.5 miles
Ridge Rd
Le Roy Ave
Hearst Ave
Gayley Rd
Highland Pl
Cyclotron Rd
South Dr
Stadium Rim Way
Optometry Ln
Seoul Hotdog
Hot Dog
Berkeley Student Cooperative - Casa...
Lower Hearst Parking Structure
North Gate Hall
Sutardja Dai Hall
Cory Hall
La Loma Parking Structure
Foothill Dining
Stern Hall
Yali's Qualcomm Café
Hearst Memorial Mining Building
O'Brien Hall
Kresge Engineering Library
East Asian Library
University House
Memorial Glade
Campus green space & student hangout
Stanley Hall
Pimentel Hall
Latimer Hall
Gilman Hall
Leconte Hall
Chou Hall
Haas School of Business
The Greek Theatre
Outdoor amphitheater for live shows
Foothill Student Housing 9
Foothill Lot
Bowles Hall
Bowles Lot
Tightwad Hill
Lot F
Big C
Building 59 - Shyh Wang Hall
Berkeley Lab Blackberry Gate
Building 15 - Berkeley Lab
ALS Offices ar
National Laboratory
Wellman Courtyard Parking
Wellman Hall
Moffitt Library
The Bancroft Library
Campanile Esplanade
University of California, Berkeley
The Faculty Club
Meredith Morgan Eye Center
UC Berkeley Optometry Clinic
Rice and Bones
Vietnamese
California Memorial Stadium
South Hall
Stephens Hall
Moses Hall
Morrison Hall
Anthony Hall
Dwinelle Hall
Dwinelle Lot
Dwinelle Annex
Sather Gate
Iconic campus gate with bronze metalwork
Valley Life Sciences Building
Department of Molecular Cell Biology
Campanile Way
Peppertree Way
Sather Rd
Carillon Rd
Eucild Walk
Soule Rd
Library Ct
University House Way
Eshleman Rd

Unweighted graphs

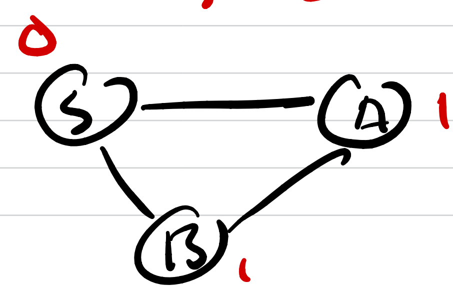


DFS:





neighbors of neighbors
(have not yet seen)
dist 2



Breadth-first search

bfs (G, s)

dist [s] = 0
$\forall u \neq s$, dist [u] = ∞

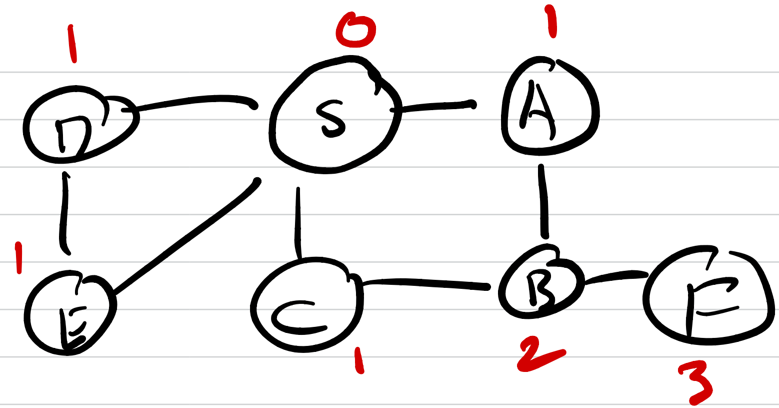
Q = {s} (queue containing s)

while Q is not empty

u = dequeue(Q)

for all v s.t. (u, v) $\in$ E
if dist [v] = ∞
enqueue (Q, v)

dist[v] = dist[u] + 1



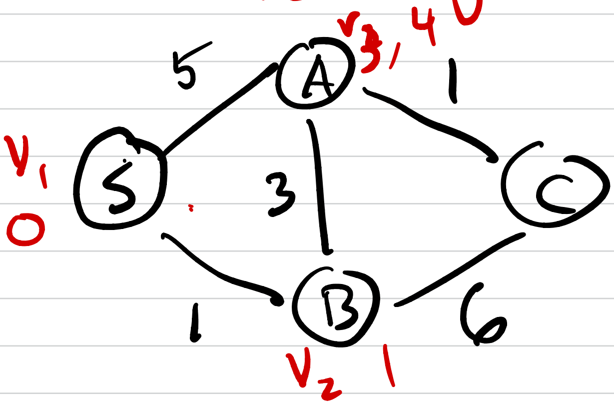
Q : ~~S~~ ~~A~~ ~~C~~ ~~D~~ ~~E~~ ~~B~~ ~~F~~

Runtime : O(n+m) time

linear, same as DFS

DFS is just BFS w/ stack

Positive lengths



Dijkstra's algorithm

1. Compute v_1 = closest vertex to s and $d(s, v_1)$
2. Compute v_2 = 2^{nd} closest to s and $d(s, v_2)$
3. " v_3 3^{rd} " "

$\vdots$

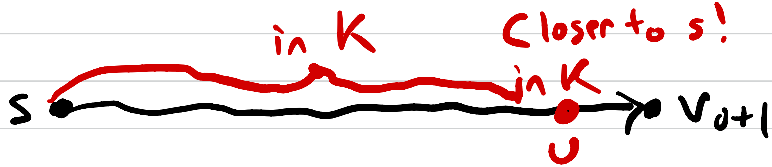
K = the set of "known" vertices at some step

$= \{v_1, \ldots, v_i\}$

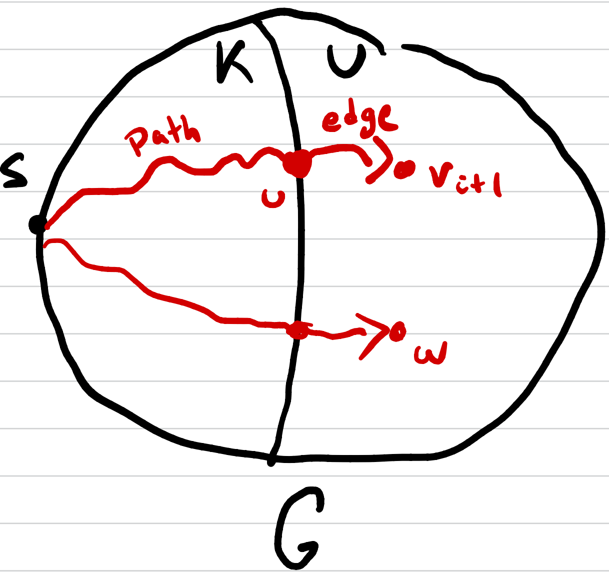
U = "unknown" vertices $\qquad U = V / K$

Q: Given $K = \{v_1, \dots, v_i\}$.

How to compute v_{i+1}? = closest vertex not in K in U



(shortest $s \to v_{i+1}$ path)



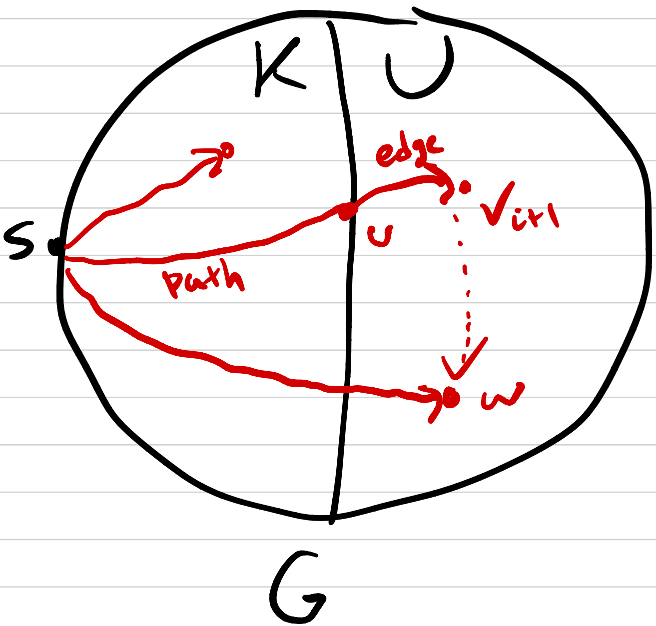
v_{i+1} = endpoint of the shortest path of this form

$$\text{dist}[v_{i+1}] = \text{dist}[u] + \ell(u, v_{i+1})$$

u minimizes $\text{dist}[u] + \ell(u, v_{i+1})$

Dijkstra has dist[v] for each $v \in V$

$$\text{dist}[v] = \begin{cases} d(s,v) & \text{if } v \in K \\ \min\limits_{u \in K} \text{dist}[u] + \ell(u,v) \end{cases}$$



After adding v_{i+1} to K

If $(v_{i+1}, w) \in E$

$$\text{dist}[w] = \min\{\text{dist}[w], \text{dist}[v_{i+1}] + \ell(v_{i+1}, w)\}$$

update(v_{i+1}, w)

dijkstra(G, ℓ, s)

dist[s] = 0

$\forall\, u \neq s$, dist[u] = ∞

U = V   (insert (u, dist[u]) $\forall$ u)

while U is not empty

Choose $u \in U$ with minimum dist[u]

Remove u from U.

(u = DeleteMin())

for each v s.t. $(u,v) \in E$

dist[v] = min(dist[v], dist[u] + ℓ(u,v))

DecreaseKey(v, dist[v])

Priority queue

Contains a set of (element, key) pairs

integer

- Insert (elem, key)
- DecreaseKey (elem, key)

  (Replaces elem's old key w/ new key)
- DeleteMin()

Dijkstra's runtime

Insert n times
DeleteMin n times
DecreaseKey m times

Implementation	Insert	DelMin	Decrease	Total
array				
binary heap				
Fibonacci heap				

Mikkel Thorup 2004: $O(n \log\log n + m)$