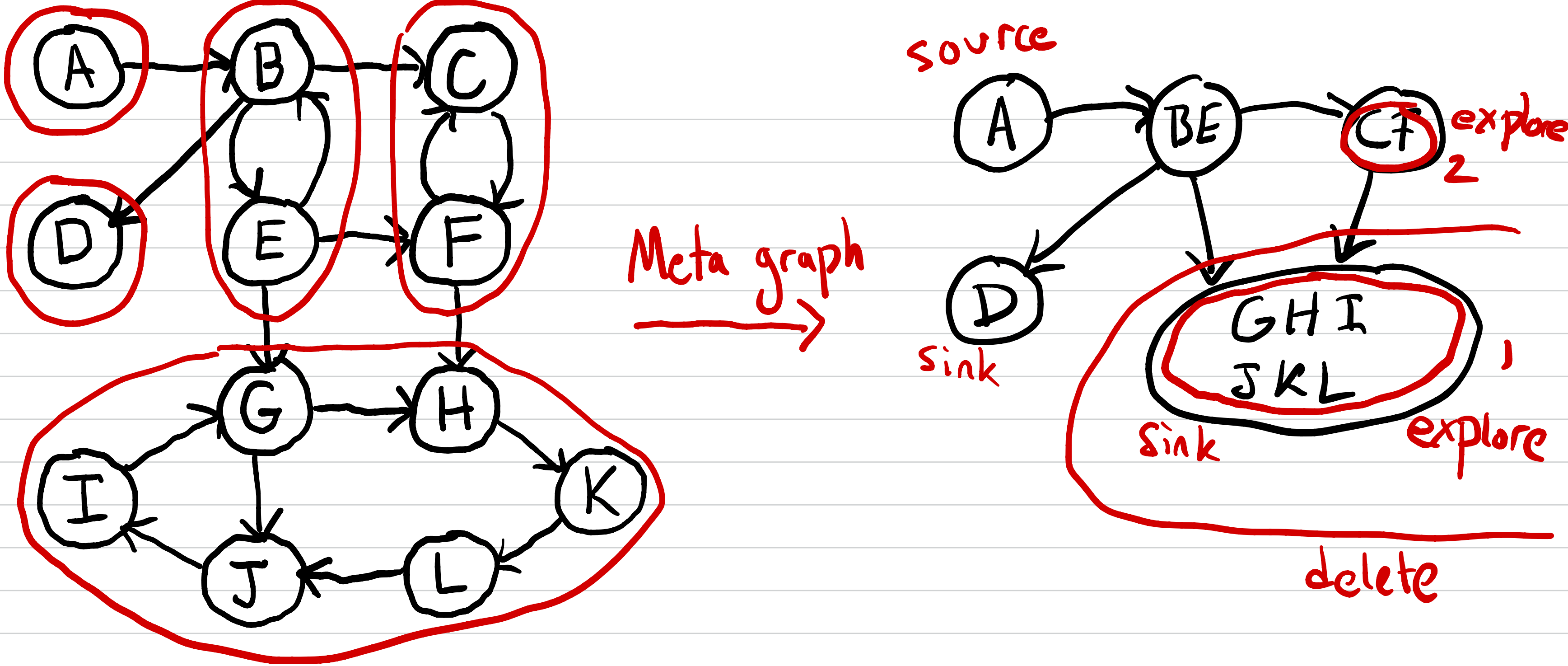
Strongly
Connected
Components



Magic algorithm: gives us vertex u in sink SCC

via DFS! (with a twist...)

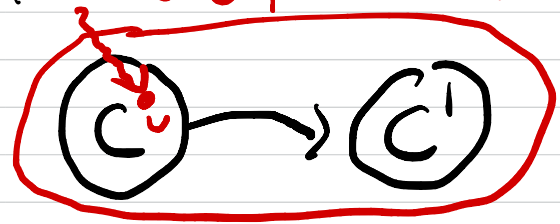
Suppose we run DFS

For all SCCs C, define finish(C) = C's highest post(v)

Claim: Let $C \to C'$ be SCCs.
Then finish(C) > finish(C').

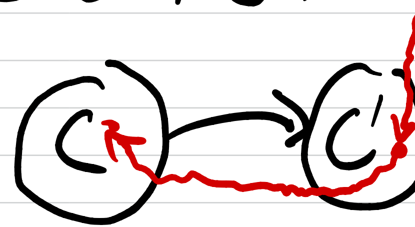
Pf: (i) Suppose DFS visits C first.
Then post(u) > finish(C').

u explores thes



(ii) Suppose DFS visits C' first.
Then only visits C after done w/ C'.

∴ all posts in C
> finish(C'). $\square$



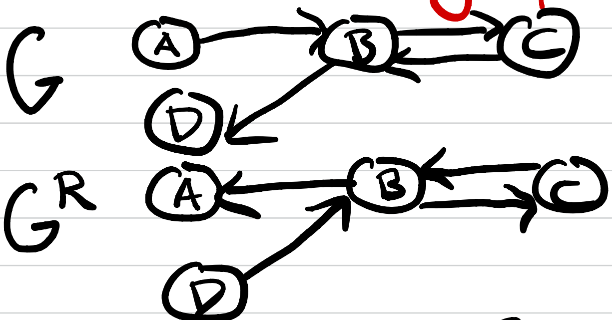
Can't happen! DAG.

Claim: The highest post(u) is in source SCC.
Assume not.

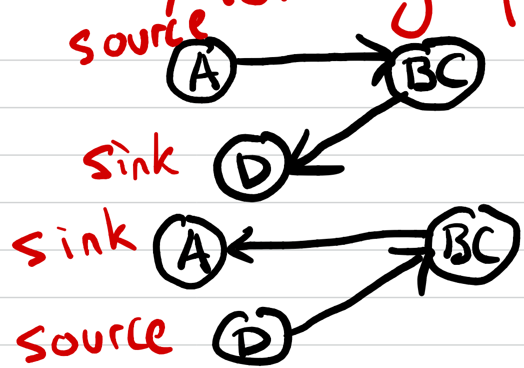
even bigger!

biggest post

The reverse graph

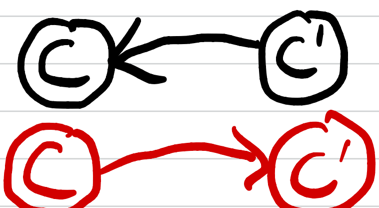


Meta graph



Claim: G and G^R have same SCCs.
In meta graphs • edges are reversed
• sources and sinks are swapped

Run DFS on G^R. Compute $post_R$ values.
u w/ highest $post_R$: • (in G^R) in source
• (in G) in sink

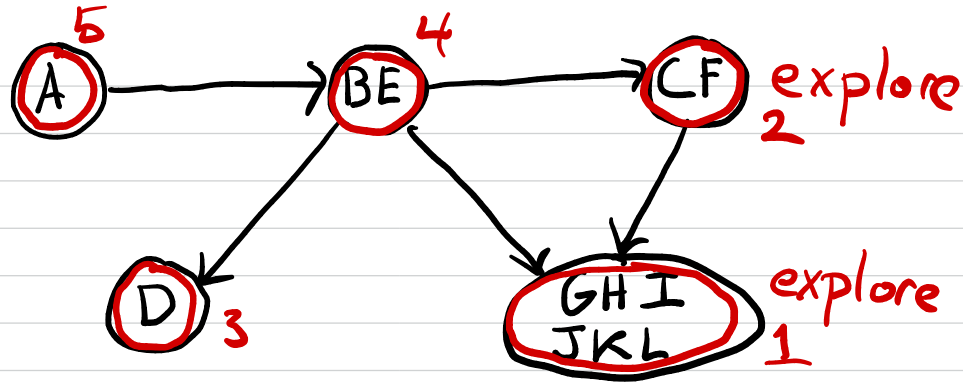
If 

(in G^R) then highest $post_R$ in C' $>$ in C

SCC algorithm

$C \to C'$

highest $post_R(u)$ in C'
> in C



explore(G,u)

```
visited[u]=true
sccnum[u]=count
for v s.t. (u,v) ∈ E
    if visited[v]=false
        explore(G,u)
```

Find SCCs(G)

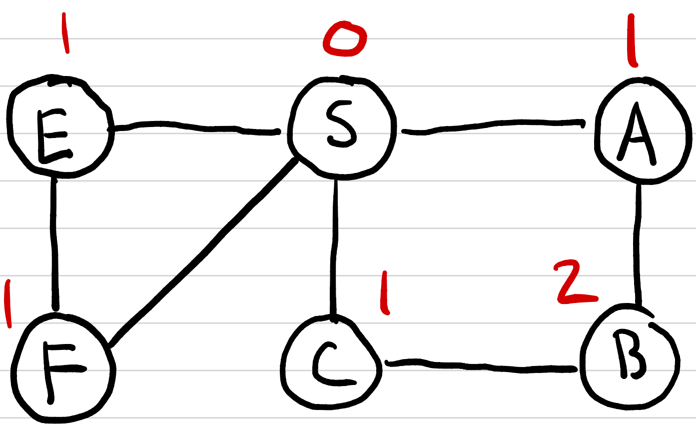
```
for all u, visited[u]=false
Run DFS on G^R
    to compute post_R
count=1
for u ∈ V (in reverse post_R order)
    if visited[u]=false
        explore(G,u)
        count=count+1
```

Paths in Graphs

Single source shortest paths (SSSP)

Input: Graph G, "source" vertex $s \in V$

Output: $\forall u \in V$, $d(s,u)$ = length of shortest path from s to u



Unweighted: all edges length 1

Breadth-first search

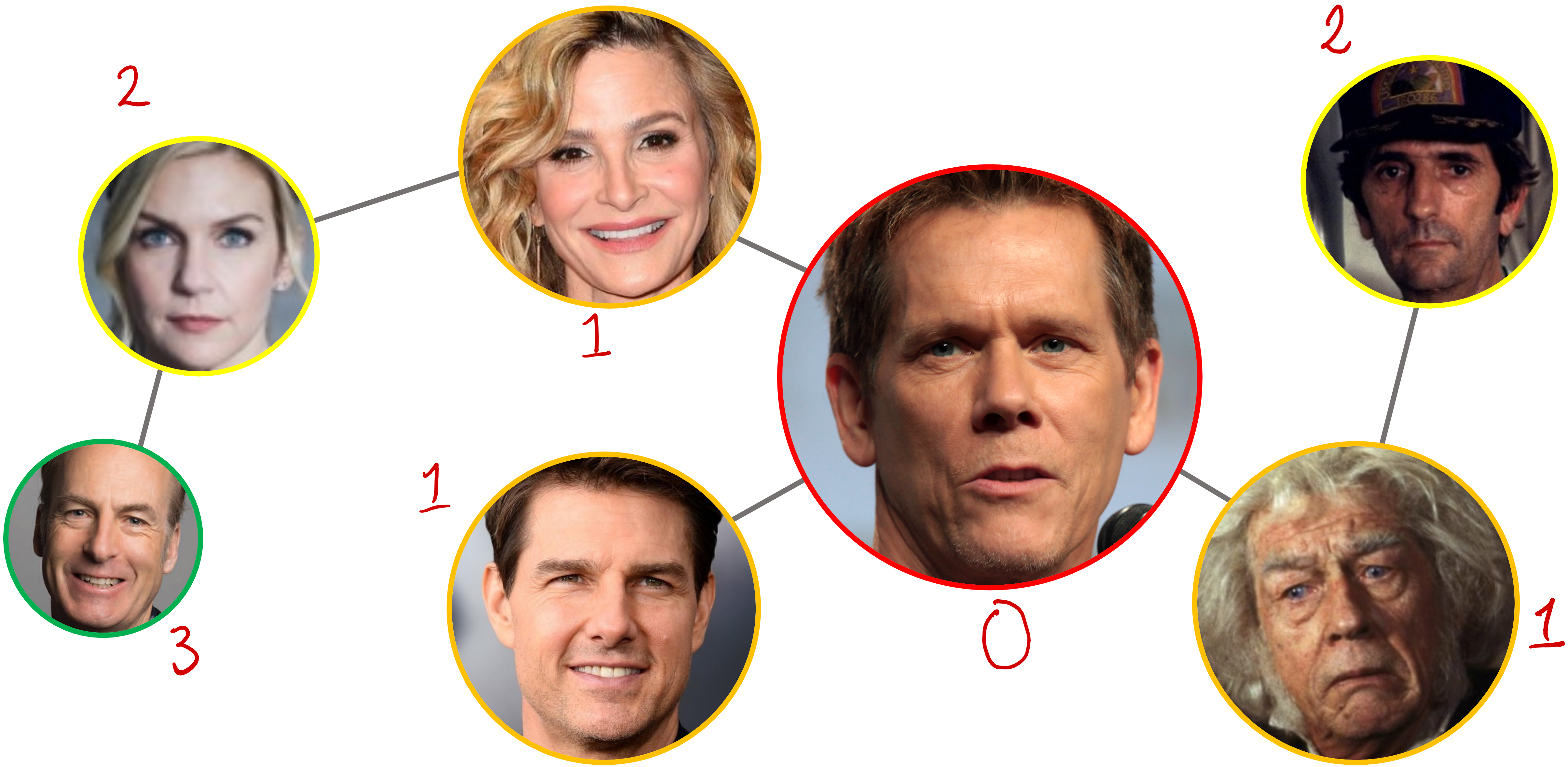
Positive lengths: $\ell : E \to \{1, 2, 3, \ldots\}$

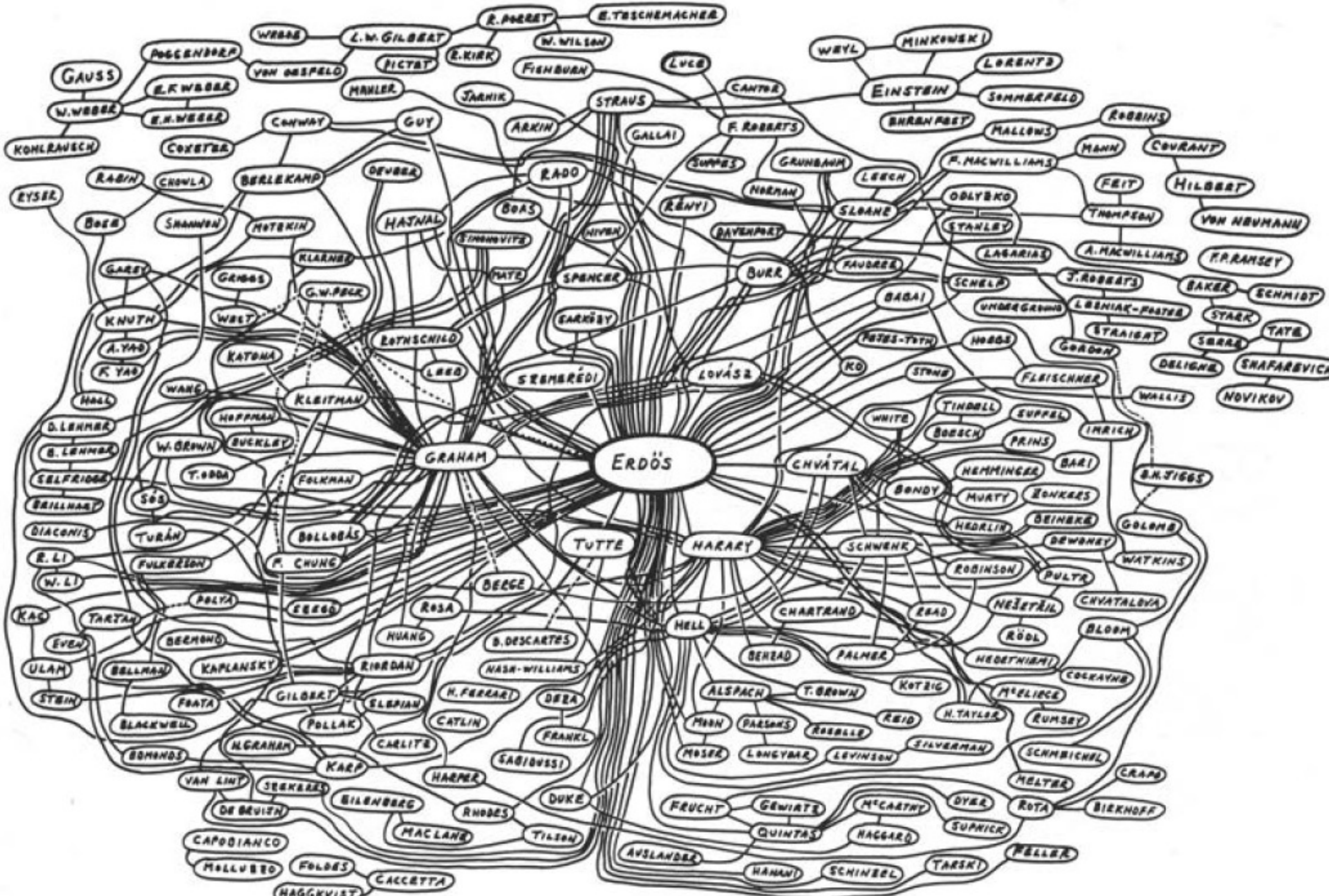
Dijkstra

Arbitrary length edges

Bellman-Ford

Application: Kevin Bacon number





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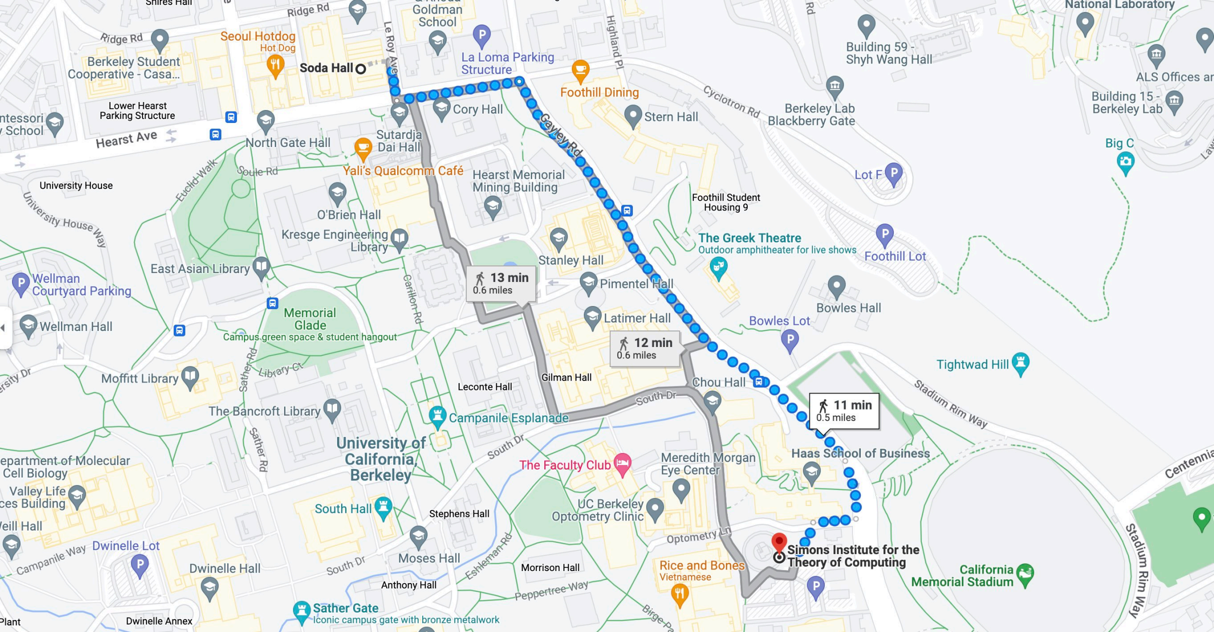
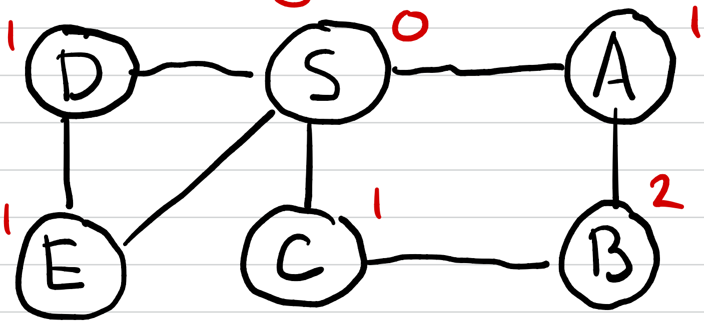
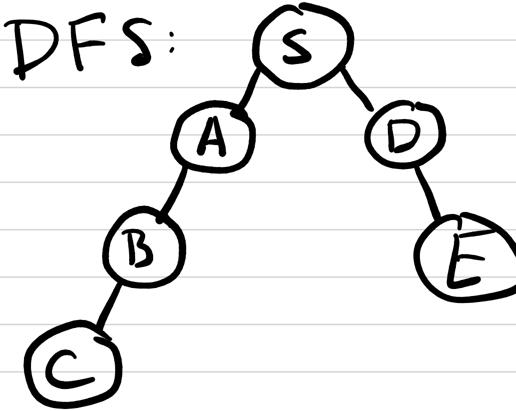


Soda Hall
Simons Institute for the Theory of Computing
13 min
0.6 miles
12 min
0.6 miles
11 min
0.5 miles
Ridge Rd
Le Roy Ave
Gayley Rd
Hearst Ave
Highland Pl
Cyclotron Rd
South Dr
Optometry Ln
Stadium Rim Way
Seoul Hotdog
Hot Dog
Berkeley Student Cooperative - Casa...
Lower Hearst Parking Structure
North Gate Hall
Cory Hall
Sutardja Dai Hall
Yali's Qualcomm Café
La Loma Parking Structure
Foothill Dining
Stern Hall
Building 59 - Shyh Wang Hall
Berkeley Lab Blackberry Gate
Building 15 - Berkeley Lab
ALS Offices
National Laboratory
Big C
Lot F
Foothill Student Housing 9
The Greek Theatre
Outdoor amphitheater for live shows
Foothill Lot
Bowles Hall
Bowles Lot
Tightwad Hill
Hearst Memorial Mining Building
O'Brien Hall
Kresge Engineering Library
Stanley Hall
Pimentel Hall
Latimer Hall
Gilman Hall
Leconte Hall
Chou Hall
Haas School of Business
University House
East Asian Library
Wellman Courtyard Parking
Wellman Hall
Memorial Glade
Campus green space & student hangout
Moffitt Library
The Bancroft Library
Campanile Esplanade
University of California, Berkeley
The Faculty Club
Meredith Morgan Eye Center
UC Berkeley Optometry Clinic
Rice and Bones
Vietnamese
California Memorial Stadium
South Hall
Stephens Hall
Moses Hall
Morrison Hall
Anthony Hall
Dwinelle Lot
Dwinelle Hall
Dwinelle Annex
Sather Gate
Iconic campus gate with bronze metalwork
Valley Life Sciences Building
Campanile Way
Peppertree Way
Eshleman Rd
Sather Rd
Carillon Rd
Library Ct
Soule Rd
Euclid Walk
University House Way

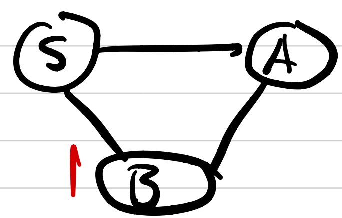
Unweighted graphs



DFS:



neighbors of neighbors
(not yet seen)
dist 2



Breadth-first search

bfs(G, s)

dist$[s] = 0$

$\forall\, u \neq s$, dist$[u] = \infty$

$Q = \{s\}$ (queue containing s)

while Q is not empty

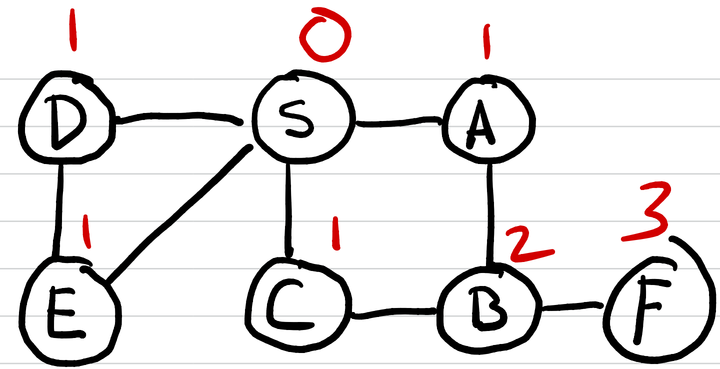
  u = dequeue(Q)

  for all v s.t. $(u,v) \in E$

    if dist$[v] = \infty$

      enqueue(Q, v)

      dist$[v]$ = dist$[u] + 1$



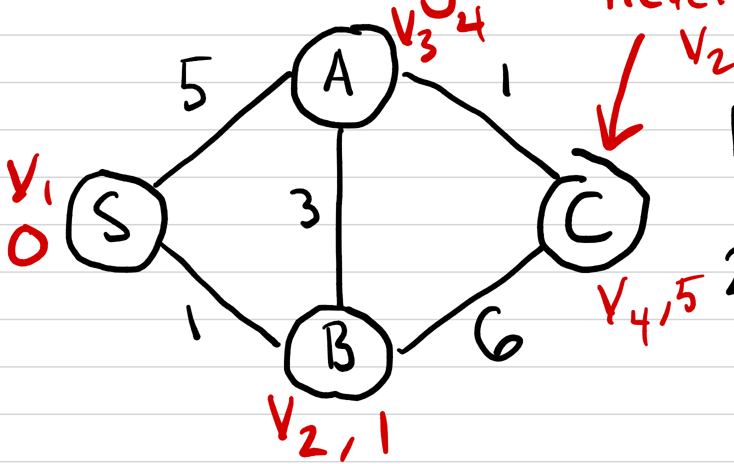
Q: ~~S~~ ~~A~~ ~~C~~ ~~D~~ ~~E~~ ~~B~~ ~~F~~

Runtime: $O(n+m)$ time

linear, same as DFS

DFS is just BFS w/ stack.

Positive lengths



Dijkstra's algorithm

1. Computing v_1 = closest vertex to s and $d(s, v_1)$
2. Compute $v_2 = 2^{\text{nd}}$ closest to s and $d(s, v_2)$

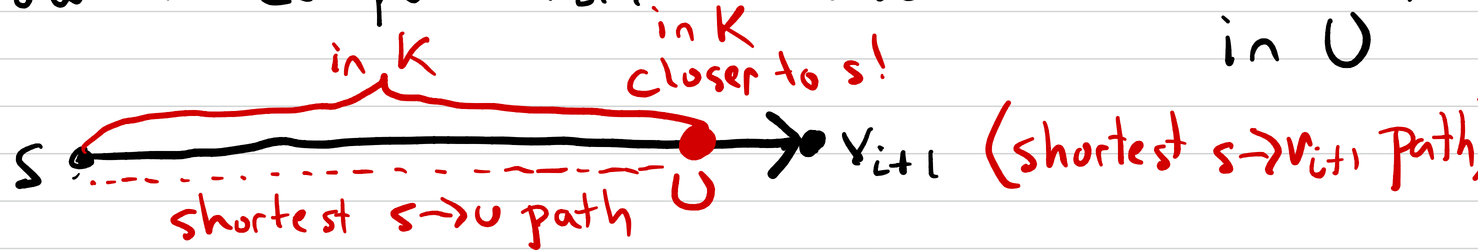
$\vdots$

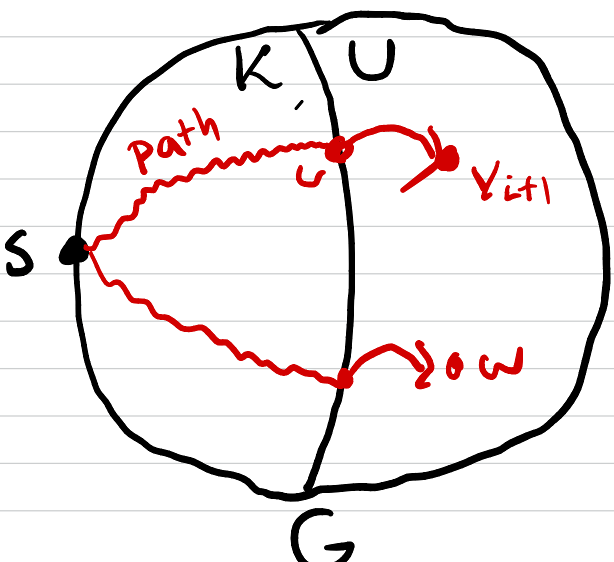
K = the set of "known" vertices at some step $= \{v_1, \ldots, v_i\}$

U = "unknown" vertices $= V \setminus K$

Q: Given $K = \{v_1, \dots, v_i\}$.

How to compute v_{i+1}? = closest vertex not in K, in U



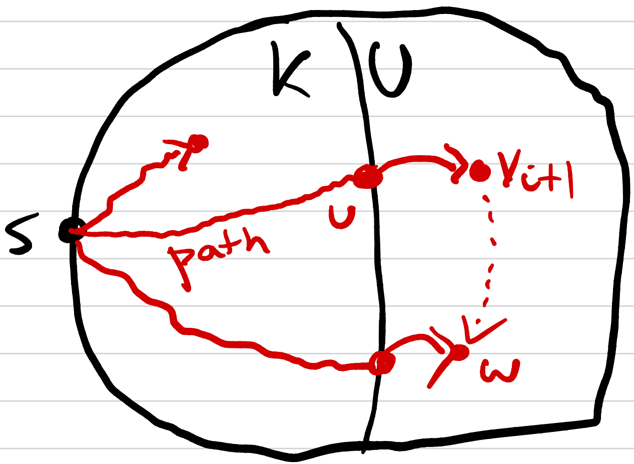


v_{i+1} = endpoint of shortest known path of this form

$$\mathrm{dist}[v_{i+1}] = \min_{u \in K} \{\mathrm{dist}[u] + \ell(u, v_{i+1})\}$$

Dijkstra has dist[v] for each $v \in V$

$$\text{dist}[v] = \begin{cases} d(s,v) & \text{if } v \in K \\ \underbrace{\min\limits_{u \in K} \left\{ \text{dist}[u] + \ell(u,v) \right\}}_{\text{help to find } v_{i+1}\,!} & \text{if } v \in U \end{cases}$$



After adding v_{i+1} to K

If $(v_{i+1}, w) \in E$

$$\text{dist}[w] = \min \left\{ \text{dist}[w],\ \text{dist}[v_{i+1}] + \ell(v_{i+1}, w) \right\}$$

dijkstra(G, ℓ, s)

$dist[s] = 0$

$\forall\, u \neq s,\ dist[u] = \infty$ n times

$U = V$ (insert $(u, dist[u])$ $\forall u$)

while U is not empty

Choose $u \in U$ with min $dist[u]$

Remove u from U.

(u = DeleteMin()) n times

for each v s.t. $(u,v) \in E$

$dist[v] = \min\{ dist[v],$

m times $dist[u] + \ell(u,v)\}$

DecreaseKey$(v, dist[v])$

Priority queue

Contains a set of (element, key) pairs

integer

- Insert (elem, key)
- DecreaseKey (elem, key)
- DeleteMin ()

Dijkstra's runtime

Insert n times
DeleteMin n times
DecreaseKey m times

Implementation	Insert	DelMin	Decrease	Total
array	$O(1)$	$O(n)$	$O(1)$	$O(n^2+m) = O(n^2)$
binary heap	$\log(n)$	$\log(n)$	$\log(n)$	$O((n+m)\log(n))$
Fib heap	$O(1)$	$\log(n)$	$O(1)$	$O(n\log n + m)$

Mikkel Thorup 2004: $O(n\log\log n + m)$