1 (★★★) Asymptotic notation

(a) For each pair of functions \( f(n) \) and \( g(n) \), state whether \( f(n) = O(g(n)) \), \( f(n) = \Omega(g(n)) \), or \( f(n) = \Theta(g(n)) \). For example, for \( f(n) = n^2 \) and \( g(n) = 2n^2 - n + 3 \), write \( f(n) = \Theta(g(n)) \).

- \( f(n) = n \) and \( g(n) = n^2 - n \)
- \( f(n) = n^2 \) and \( g(n) = n^2 + n \)
- \( f(n) = 8n \) and \( g(n) = n \log n \)
- \( f(n) = 2^n \) and \( g(n) = n^2 \)
- \( f(n) = 3^n \) and \( g(n) = 2^n \)

(b) For each of the following, state the order of growth using \( \Theta \) notation, e.g. \( f(n) = \Theta(n) \).

- \( f(n) = 50 \)
- \( f(n) = n^2 - 2n + 3 \)
- \( f(n) = n + \cdots + 3 + 2 + 1 \)
- \( f(n) = n^{100} + 1.01^n \)
- \( f(n) = n^{1.1} + n \log n \)
- \( f(n) = (g(n))^2 \) where \( g(n) = \sqrt{n} + 5 \)
2 Asymptotic Bound Practice

Prove that for any $\epsilon > 0$ we have $\log x = O(x^\epsilon)$.

3 Bounding Sums

Let $f(\cdot)$ be a function. Consider the equality

$$\sum_{i=1}^{n} f(i) = \Theta(f(n)),$$

Give a function $f_1$ such that the equality holds, and a function $f_2$ such that the equality does not hold.