CS 170 DIS 1

Released on 2019-01-22

1 (★★★) Asymptotic notation

(a) For each pair of functions $f(n)$ and $g(n)$, state whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. For example, for $f(n) = n^2$ and $g(n) = 2n^2 - n + 3$, write $f(n) = \Theta(g(n))$.

- $f(n) = n$ and $g(n) = n^2 - n$
- $f(n) = n^2$ and $g(n) = n^2 + n$
- $f(n) = 8n$ and $g(n) = n \log n$
- $f(n) = 2^n$ and $g(n) = n^2$
- $f(n) = 3^n$ and $g(n) = 2^{2n}$

(b) For each of the following, state the order of growth using Θ notation, e.g. $f(n) = \Theta(n)$.

- $f(n) = 50$
- $f(n) = n^2 - 2n + 3$
- $f(n) = n + \cdots + 3 + 2 + 1$
- $f(n) = n^{100} + 1.01^n$
- $f(n) = n^{1.1} + n \log n$
- $f(n) = (g(n))^2$ where $g(n) = \sqrt{n} + 5$

2 Asymptotic Bound Practice

Prove that for any $\epsilon > 0$ we have $\log x = O(x^\epsilon)$.

3 Bounding Sums

Let $f(\cdot)$ be a function. Consider the equality

$$\sum_{i=1}^{n} f(i) \;=\; \Theta(f(n)),$$

Give a function f_1 such that the equality holds, and a function f_2 such that the equality does not hold.