CS 170 DIS 02

Released on 2019-01-28

1 Squaring vs multiplying: matrices

The square of a matrix A is its product with itself, AA.

(a) Show that five multiplications are sufficient to compute the square of a 2×2 matrix.

(b) What is wrong with the following algorithm for computing the square of an $n \times n$ matrix?

"Use a divide-and-conquer approach as in Strassen's algorithm, except that instead of getting 7 subproblems of size $n/2$, we now get 5 subproblems of size $n/2$ thanks to part (a). Using the same analysis as in Strassen's algorithm, we can conclude that the algorithm runs in $\Theta(n^{\log_2 5})$ time."

(c) In fact, squaring matrices is no easier than multiplying them. Show that if $n \times n$ matrices can be squared in $\Theta(n^c)$ time, then any $n \times n$ matrices can be multiplied in $\Theta(n^c)$ time.

2 Recurrence Relations

Solve the following recurrence relations and give a Θ bound for each of them.

(a) (i) $T(n) = 3T(n/4) + 4n$

(ii) $T(n) = 45T(n/3) + .1n^3$

(iii) $T(n) = T(n-1) + c^n$, where c is a constant.

(b) $T(n) = 2T(\sqrt{n}) + 3$, and $T(2) = 3$. (Hint: this means the recursion tree stops when the problem size is 2)

3 Complex numbers review

(a) Write each of the following numbers in the form $\rho(\cos\theta + i\sin\theta)$ (for real ρ and θ):

(i) $-\sqrt{3} + i$

(ii) The three third roots of unity

(iii) The sum of your answers to the previous item

(b) Let sqrt(x) represent one of the complex square roots of x, so that $(\text{sqrt}(x))^2 = x$. What are the possible values of sqrt(sqrt(-1))?

You can use any notation for complex numbers, e.g., rectangular, polar, or complex exponential notation.

4 Practice with Polynomial Multiplication with FFT

(a) Suppose that you want to multiply the two polynomials $x+1$ and x^2+1 using the FFT. Choose an appropriate power of two, find the FFT of the two sequences, multiply the results componentwise, and compute the inverse FFT to get the final result.

(b) Repeat for the pair of polynomials $1 + x + 2x^2$ and $2 + 3x$.