CS 170 Dis 03

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1 Cubed Fourier

(a) Cubing the 9^{th} roots of unity gives the 3^{rd} roots of unity. Next to each of the third roots below, write down the corresponding 9^{th} roots which cube to it. The first has been filled for you. *We will use ω_9 to represent the primitive 9^{th} root of unity, and ω_3 to represent the primitive 3^{rd} root.*

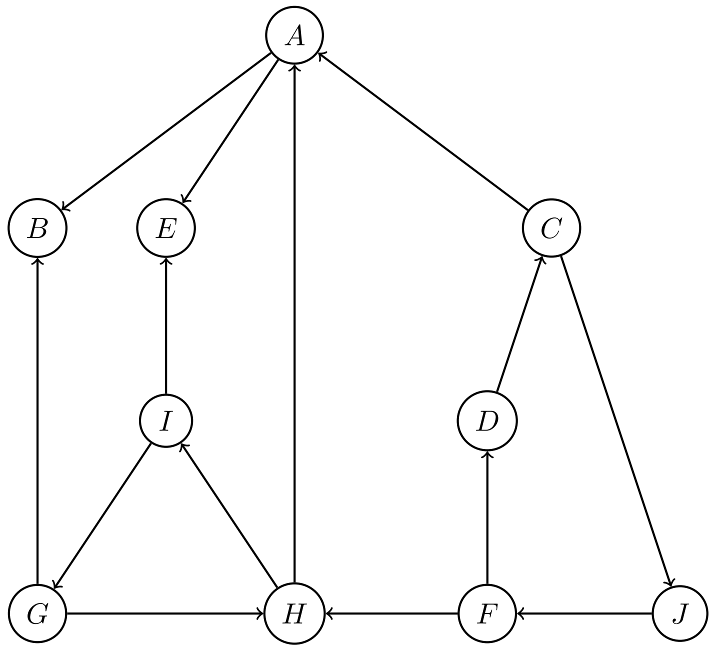
$\omega_3^0 : \omega_9^0, \quad ,$

$\omega_3^1 : \quad , \quad ,$

$\omega_3^2 : \quad , \quad ,$

(b) You want to run FFT on a degree-8 polynomial, but you don't like having to pad it with 0s to make the (degree+1) a power of 2. Instead, you realize that 9 is a power of 3, and you decide to work directly with 9th roots of unity and use the fact proven in part (a). Say that your polynomial looks like $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_8x^8$. **How do you split $P(x)$ to use the fact proven in part (a) to your advantage?** Provide either the polynomial, or explain how the vector can be divided to recurse on. *Recall that for the FFT algorithm shown in the book, we split a given polynomial $Q(x) = A_e(x^2) + xA_o(x^2)$, and we define what $A_e(x^2)$ and $A_o(x^2)$ are. Correspondingly, in lecture you saw the $\vec{a}$ split into $\vec{a}_{even}$ and $\vec{a}_{odd}$.*

2 Graph Traversal



(a) For the directed graph above, perform DFS starting from vertex A, breaking ties alphabetically. As you go, label each node with its pre- and post-number, and mark each edge as **T**ree, **B**ack, **F**orward or **C**ross.

(b) What are the strongly connected components of the above graph?

(c) Draw the DAG of the strongly connected components of the graph.

3 Finding Clusters

We are given a directed graph $G = (V, E)$, where $V = \{1, \ldots, n\}$, i.e. the vertices are integers in the range 1 to n. For every vertex i we would like to compute the value $m(i)$ defined as follows: $m(i)$ is the smallest j such that vertex j is reachable from vertex i. (As a convention, we assume that i is reachable from i.) Show that the values $m(1), \ldots, m(n)$ can be computed in $O(|V| + |E|)$ time.