CS 170 Discussion 4 Reference Sheet

Graphs Cheatsheet

Depth First Search (DFS)

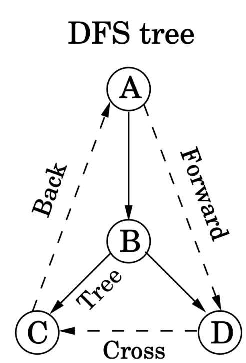
```
def dfs(G, s):
    def explore_recursive(G, v):
        visited(v) = true
        previsit(v) # set the pre-order of v
        for each edge (v, u) in E:
            if not visited(u):
                explore_recursive(G, u)
        postvisit(v) # set the post-order of v

    def explore_iterative(G, v):
        st = stack()
        st.push(v)

        while st is not empty:
            u = st.pop()
            visited(u) = true

            for each edge (u, w) in E:
                if not visited(w):
                    st.push(w)

    # depending on how you want to DFS, you can use
    # either explore_recursive or explore_iterative below
    explore(G, s)
    for all v in V:
        if not visited(v):
            explore(G, v)
```

↪ **Runtime of DFS:** $O(|V| + |E|)$

DFS Tree/Forest: the tree/forest produced by the edges traversed during a given DFS

Edge Types:

- *Tree Edge:* leads to child; part of the DFS Tree/Forest
- *Forward Edge:* leads to a non-child descendant
- *Back Edge:* leads to an ancestor
- *Cross Edge:* leads to a node that's neither a descendant nor an ancestor



Edge Type based on Pre/Post-orders: an edge $(u, v) \in E$ is a:

- *Tree or Forward Edge* if $\text{pre}(u) < \text{pre}(v) < \text{post}(v) < \text{post}(u)$
- *Back Edge* if $\text{pre}(v) < \text{pre}(u) < \text{post}(u) < \text{post}(v)$
- *Cross Edge* if $\text{pre}(v) < \text{post}(v) < \text{pre}(u) < \text{post}(u)$

Topological Sort (Graph Linearization)

```
# Returns the topological order of vertices in G (acyclic)
def topo_sort(G):
    topo_order = []
    def explore(G, v):
        visited(v) = true
        for each edge (v, u) in E:
            if not visited(u):
                explore(G, u)
        topo_order.append(v) # note that topological order is reverse post-order!

    s = any arbitrary node in G
    explore(s)
    for all v in V:
        if not visited(v):
            explore(G, v)

    return topo_order[::-1]
```

Breadth First Search (BFS)

```
def bfs(G, s):
    q = queue()
    q.push(v)

    while q is not empty:
        v = q.pop()
        visited(v) = true

        for each edge (v, u) in E:
            if not visited(u):
                q.push(u)
```

$\hookrightarrow$ **Runtime of BFS:** $O(|V| + |E|)$

Strongly Connected Components

A **strongly connected component** of G is a subset of vertices in which there is a path from every vertex to every other vertex.

Kosaraju's Algorithm

Given a graph $G = (V, E)$, we can find all the SCCs as follows:

1. Run DFS on G^{rev} to get the post-order values of all vertices $v \in V$; i.e. we compute $\text{post}^{\text{rev}}(v)$ for all $v \in V$.
2. Run DFS on G starting at the vertex with the highest post-order in G^{rev} (that is unvisited), which must belong in the sink SCC of G. Throughout this DFS, we label each traversed vertex as part of the current SCC.
3. Repeat steps 2-3 until we've labeled all SCCs.

$\hookrightarrow$ **Runtime of Kosaraju's:** $O(|V| + |E|)$

Dijkstra's Algorithm

$\hookrightarrow$ Given a graph G with non-negative edge weights $w(\cdot)$, finds the shortest path lengths from s to all vertices

```
def dijkstra(G, s):
    for all v in V:
        dist(v) = infinity # distances
        par(v) = none # parents in shortest paths tree

    dist(s) = 0
    h = min_heap() # priority according to distance
    h.insert((s, 0))

    while h is not empty:
        v = h.delete_min()
        for each edge (v, u) in E:
            if dist(u) > dist(v) + w(v, u):
                dist(u) = dist(v) + w(v, u)
                par(u) = v
                h.decrease_key(u) # sets priority of u to be the updated dist(u)

    return dist, par
```

$\hookrightarrow$ **Runtime of Dijkstra's:**

- $O((|E| + |V|) \log |V|)$ using a binary min-heap
- $O(|E| + |V| \log |V|)$ using a fibonacci min-heap