CS 170 DIS 08

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1 A HeLPful Introduction

Find necessary and sufficient conditions on real numbers a and b under which the linear program

$$\begin{aligned} \max\ & x + y \\ ax + by &\leq 1 \\ x, y &\geq 0 \end{aligned}$$

(a) Is infeasible.

(b) Is unbounded.

(c) Has a unique optimal solution.

2 Standard Form LP

Recall that any Linear Program can be reduced to a more constrained *standard form* where all variables are nonnegative, the constraints are given by equations and the objective is that of minimizing a cost function. For each of the subparts, what system of variables, constraints, and objectives would be equivalent to the following:

(a) Max Objective: $\max c^\top x$

(b) Min Max Objective: $\min \max(y_1, y_2)$

(c) Upper Bound on Variable: $x_1 \leq b_1$

(d) Lower Bound on Variable: $x_2 \geq b_2$

(e) Bounded Variable: $b_2 \leq x_3 \leq b_1$

(f) Inequality Constraint: $x_1 + x_2 + x_3 \leq b_3$

(g) Unbounded Variable: $x_4 \in R$

3 Job Assignment

There are I people available to work J jobs. The value of person i working 1 day at job j is a_{ij} for $i = 1, \dots, I$ and $j = 1, \dots, J$. Each job is completed after the sum of the time of all workers spend on it add up to be 1 day, though partial completion still has value (i.e. person i working c portion of a day on job j is worth $a_{ij}c$). The problem is to find an optimal assignment of jobs for each person for one day such that the total value created by everyone working is optimized. No additional value comes from working on a job after it has been completed.

(a) What variables should we optimize over? I.e. in the canonical linear programming definition, what is x?

(b) What are the constraints we need to consider? Hint: there are three major types.

(c) What is the maximization function we are seeking?