

CS 170 DIS 09

Released on 2018-10-22

1 Provably Optimal

Consider the following linear program:

$$\begin{aligned} \max \quad & x_1 - 2x_3 \\ & x_1 - x_2 \leq 1 \\ & 2x_2 - x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

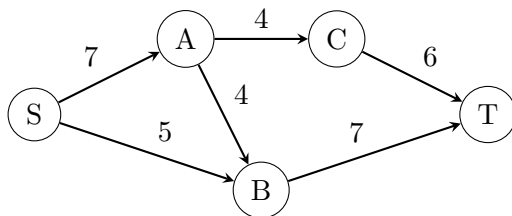
For the linear program above,

(a) First compute the dual of the above linear program

(b) show that the solution $(x_1, x_2, x_3) = (3/2, 1/2, 0)$ is optimal **using its dual**. You do not have to solve for the optimum of the dual. (*Hint*: Recall that any feasible solution of the dual is an upper bound on any feasible solution of the primal)

2 Residual in graphs

Consider the following graph with edge capacities as shown:



- (a) Consider pushing 4 units of flow through $S \rightarrow A \rightarrow C \rightarrow T$. Draw the residual graph after this push.
- (b) Compute a maximum flow of the above graph. Find a minimum cut. Draw the residual graph of the maximum flow.

3 Verifying a max-flow

Suppose someone presents you with a solution to a max-flow problem on some network. Give a *linear* time algorithm to determine whether the solution does indeed give a maximum flow.