CS 170 DIS 14

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1 A Reduction Warm-up

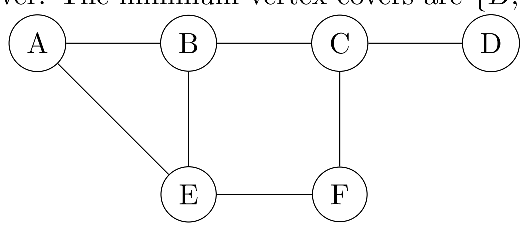
In the Rudrata path problem (aka the Hamiltonian Path Problem), we are given a graph G and want to find if there is a path in G that uses each vertex exactly once.

Is the following argument correct? Please justify your answer.

We will show that Undirected Rudrata Path can be reduced to Longest Path in a DAG. Given a graph G, use DFS to find a traversal of G and assign directions to all the edges in G based on this traversal (i.e. edges will point in the same direction they were traversed and back edges will be omitted). This gives a DAG. If the longest path in this DAG has $|V| - 1$ edges then there is a Rudrata path in G since any simple path with $|V| - 1$ edges must visit every vertex.

2 Reducing Vertex Cover to Set Cover

In the minimum vertex cover problem, we are given an undirected graph $G = (V, E)$ and asked to find the smallest set $U \subseteq V$ that "covers" the set of edges E. In other words, we want to find the smallest set U such that for each $(u, v) \in E$, either u or v is in U (U is not necessarily unique). For example, in the following graph, $\{A, E, C, D\}$ is a vertex cover, but not a minimum vertex cover. The minimum vertex covers are $\{B, E, C\}$ and $\{A, E, C\}$.



Recall the following definition of the minimum Set Cover problem: Given a set U of elements and a collection $S_1, \ldots, S_m$ of subsets of U, what is the smallest collection of these sets whose union equals U? So, for example, given $U := \{a, b, c, d\}$, $S_1 := \{a, b, c\}$, $S_2 := \{b, c\}$, and $S_3 := \{c, d\}$, a solution to the problem is the collection of S_1 and S_3.

Give an efficient reduction from the Minimum Vertex Cover Problem to the Minimum Set Cover Problem.