CS 170 Homework 5

Due 3/1/2022, at 10:00 pm (grace period until 11:59pm)

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write “none”.

2 Copper Pipes

Bubbles has a copper pipe of length n inches and an array of nonnegative integers that contains prices of all pieces of size smaller than n. He wants to find the maximum value he can make by cutting up the pipe and selling the pieces. For example, if length of the pipe is 8 and the values of different pieces are given as following, then the maximum obtainable value is 22 (by cutting in two pieces of lengths 2 and 6).

<table>
<thead>
<tr>
<th>length</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>1</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>17</td>
<td>17</td>
<td>20</td>
</tr>
</tbody>
</table>

Give a dynamic programming algorithm so Bubbles can find the maximum obtainable value given any pipe length and set of prices. Clearly describe your algorithm, prove its correctness and runtime.

3 Egg Drop

You are given k identical eggs and an n story building. You need to figure out the highest floor $\ell \in \{0, 1, 2, \ldots n\}$ that you can drop an egg from without breaking it. Each egg will never break when dropped from floor $\ell$ or lower, and always breaks if dropped from floor $\ell + 1$ or higher. ($\ell = 0$ means the egg always breaks). Once an egg breaks, you cannot use it any more. However, if an egg does not break, you can reuse it.

Let $f(n, k)$ be the minimum number of egg drops that are needed to find $\ell$ (regardless of the value of $\ell$).

(a) Find $f(1, k)$, $f(0, k)$, $f(n, 1)$, and $f(n, 0)$.

(b) Find a recurrence relation for $f(n, k)$. Hint: Whenever you drop an egg, call whichever of the egg breaking/not breaking leads to more drops the “worst-case event”. Since we need to find $\ell$ regardless of its value, you should assume the worst-case event always happens.