

CS 170 HW 10

Due 2019-11-09, at 10:00 pm

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write none.

In addition, we would like to share correct student solutions that are well-written with the class after each homework. Are you okay with your correct solutions being used for this purpose? Answer “Yes”, “Yes but anonymously”, or “No”

2 Exam Re-Attempt

For this problem, please re-attempt the midterm problems that you either did not answer (including the options for Questions 3 and 4 that you did not attempt) or did not get the right answer for on your first attempt.

After attempting each problem, read the solution. If you got it wrong, after reading the solution, close the solutions and summarize what you understood about the answer.

3 Graph Coloring Problem

An undirected graph $G = (V, E)$ is k -colorable if we can assign every vertex a color from the set $1, \dots, k$, such that no two adjacent vertices have the same color. In the k -coloring problem, we are given a graph G and want to output “Yes” if it is k -colorable and “No” otherwise.

- Show how to reduce the 2-coloring problem to the 3-coloring problem. That is, describe an algorithm that takes a graph G and outputs a graph G' , such that G' is 3-colorable if and only if G is 2-colorable. To prove the correctness of your algorithm, describe how to construct a 3-coloring of G' from a 2-coloring of G and vice-versa. (No runtime analysis needed).
- The 2-coloring problem has a $O(|V| + |E|)$ -time algorithm. Does the above reduction imply an efficient algorithm for the 3-coloring problem? If yes, what is the runtime of the resulting algorithm? If no, justify your answer.

4 Some Sums

Given an array $A = [a_1, a_2, \dots, a_n]$ of nonnegative integers, consider the following problems:

- Partition:** Determine whether there is a subset $P \subseteq [n]$ ($[n] := \{1, 2, \dots, n\}$) such that
$$\sum_{i \in P} a_i = \sum_{j \in [n] \setminus P} a_j$$
- Subset Sum:** Given some integer t , determine whether there is a subset $P \subseteq [n]$ such that
$$\sum_{i \in P} a_i = t$$

3 **Knapsack:** Given some set of items each with weight w_i and value v_i , and fixed numbers W and V , determine whether there is some subset $P \subseteq [n]$ such that $\sum_{i \in P} w_i \leq W$ and $\sum_{i \in P} v_i \geq V$

For each of the following clearly describe your reduction and justify its correctness.

- (a) Find a linear time reduction from SUBSET SUM to PARTITION.
- (b) Find a linear time reduction from SUBSET SUM to KNAPSACK.