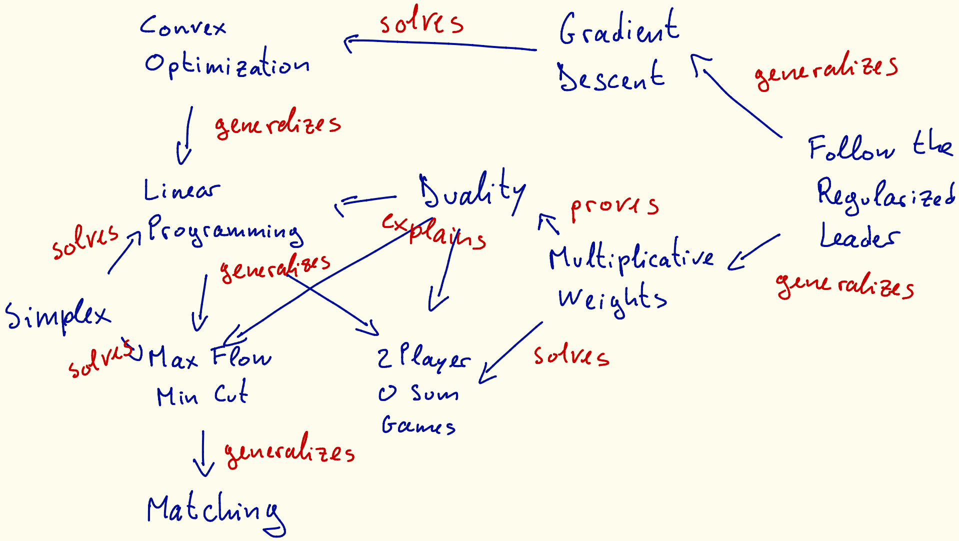
LINEAR PROGRAMMING

INTRODUCTION



Convex Optimization
solves
Gradient Descent
generalizes
generalizes
Follow the Regularized Leader
Linear Programming
Duality
proves
solves
explains
generalizes
Multiplicative Weights
generalizes
Simplex
solves
Max Flow Min Cut
2 Player 0 Sum Games
solves
generalizes
Matching

Linear Programming

Classic $4 a gallon profit

Premium $ 8 a gallon profit

Each week can brew

$\leq$ 2,000 gallons of classic

$\leq$ 2,000 gallons of premium

$\leq$ 3,000 gallons in total

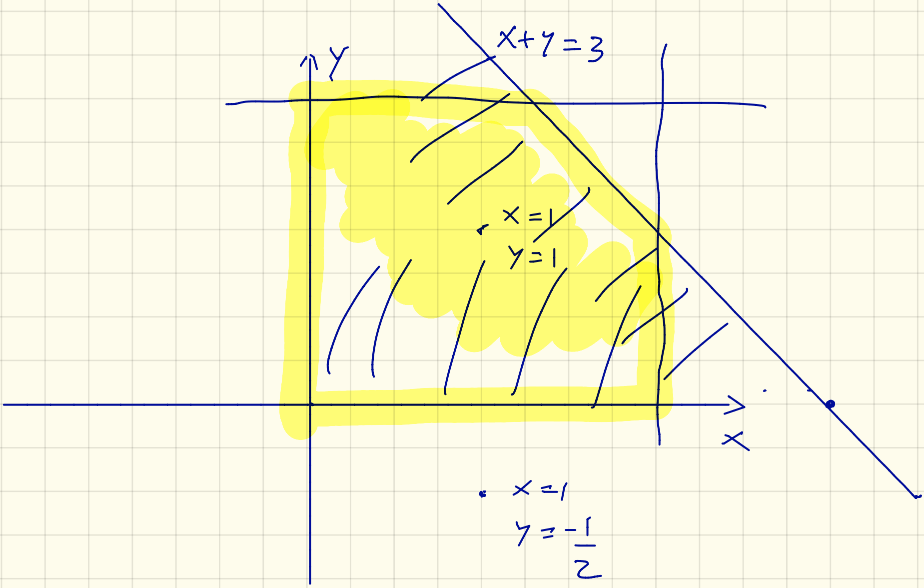
x = production per week of classic in 1,000s of gallons

y = production per week of premium in 1,000s of gallons

maximize $4x + 8y$

subject to

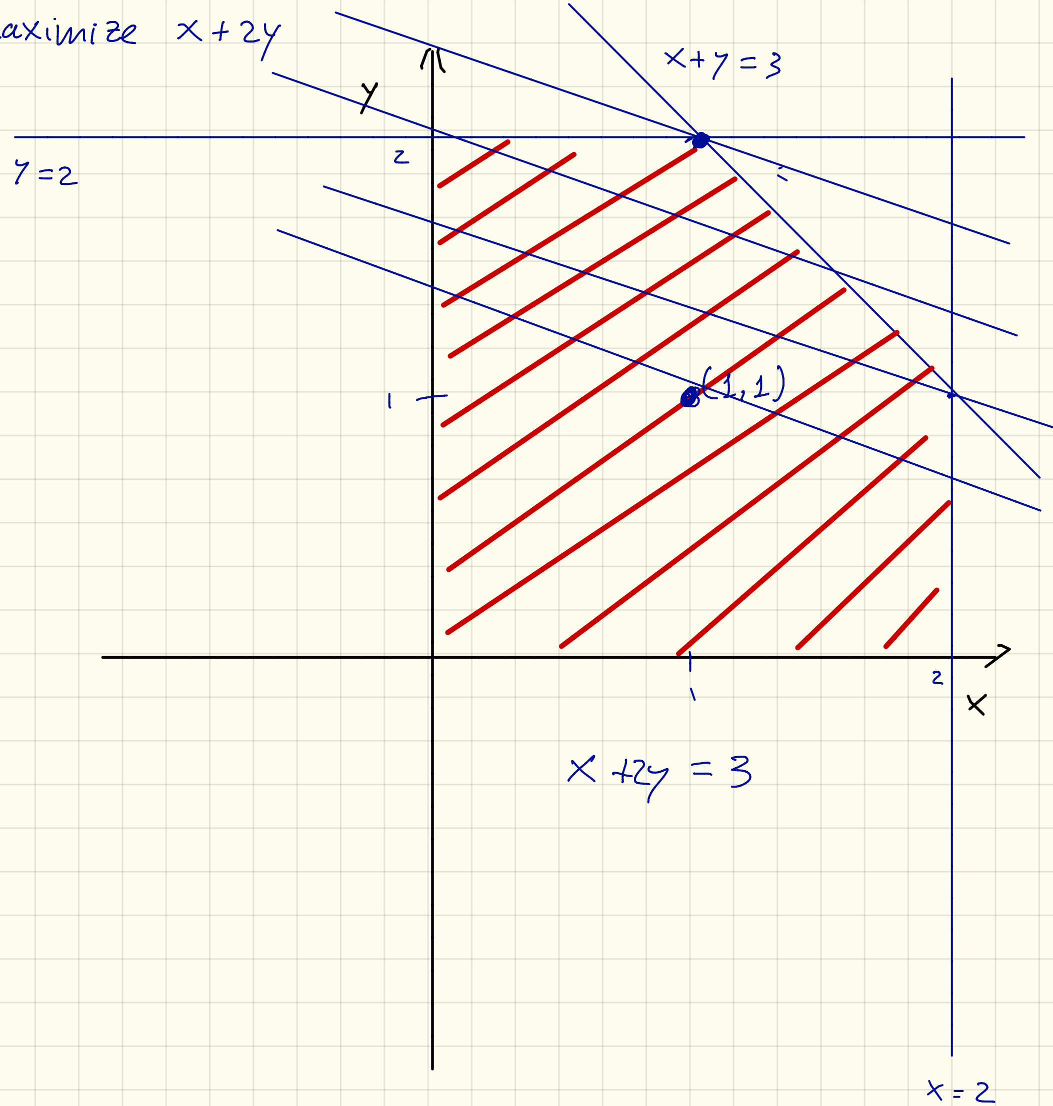
$$x \leq 2$$
$$y \leq 2$$
$$x + y \leq 3$$
$$x \geq 0$$
$$y \geq 0$$



$x + y \le 3$ $\qquad$ $y \geq 0$ $\quad$ $y \le 2$

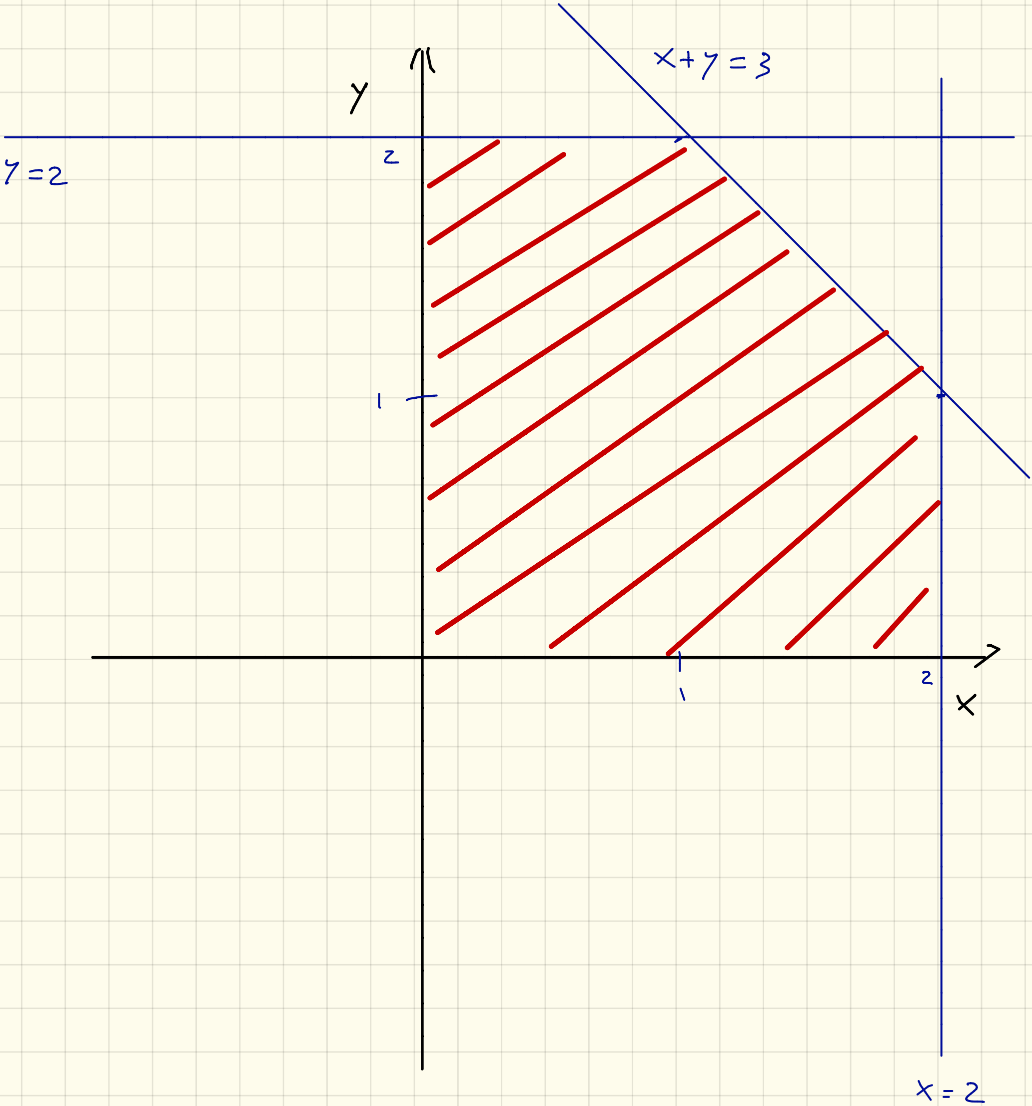
$x \geq 0$ $\qquad$ $x \le 2$

maximize $x + 2y$



Feasible region of

$$\begin{aligned} x + y &\leq 3 \\ x &\leq 2 \\ y &\leq 2 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

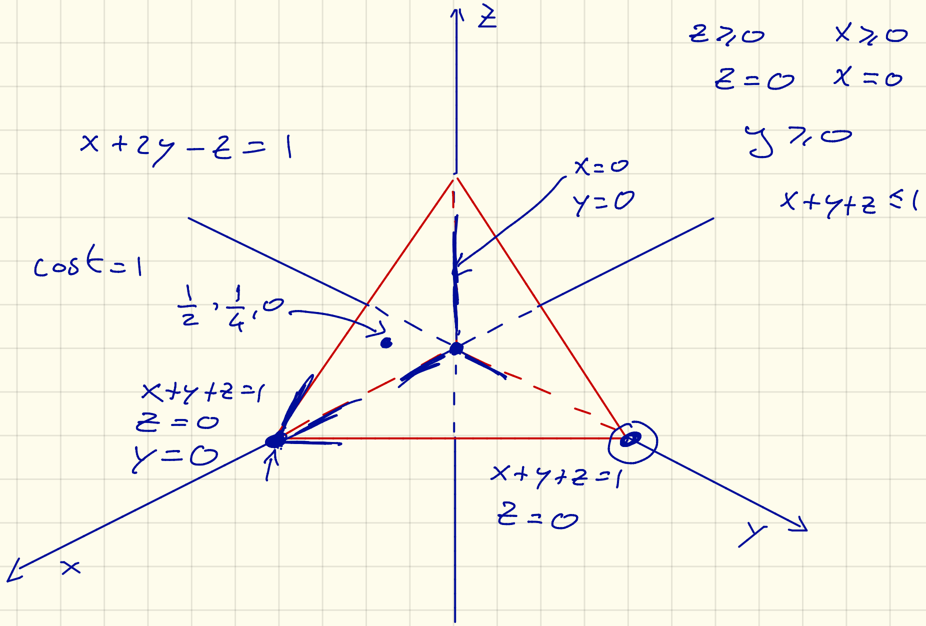


x+y=3
y
y=2
2
1
1
2
x
x=2

Maximize $x + 2y - z$

subject to

$$x + y + z \leq 1$$
$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$



max ___ -

subject to

$$x_1 \geq 0$$
$$x_1 \leq 1$$
$$x_2 \geq 0$$
$$x_2 \leq 1$$
$$\vdots$$
$$x_n \geq 0$$
$$x_n \leq 1$$

cost function

$$\text{maximize} \quad c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

constraints

$$a^1_1 x_1 + a^1_2 x_2 + \cdots + a^1_n x_n \leq b_1$$

$$a^2_1 x_1 + a^2_2 x_2 + \cdots + a^2_n x_n \leq b_2$$

$$\vdots$$

$$a^m_1 x_1 + a^m_2 x_2 + \cdots + a^m_n x_n \leq b_m$$

$$x_1 \geq 0$$
$$x_2 \geq 0$$
$$\vdots$$
$$x_n \geq 0$$

$$\boxed{\begin{aligned} &\max \ \vec{c}^T \vec{x} \\ &\text{s.t.} \\ &\quad A\vec{x} \leq \vec{b} \\ &\quad \vec{x} \geq \vec{0} \end{aligned}}$$

$$\text{maximize} \quad [c_1 \cdots c_n] \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

s.t.

$$\begin{pmatrix} a^1_1 & \cdots & a^1_n \\ a^m_1 & \cdots & a^m_n \end{pmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \leq \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \geq \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\max \quad x_1 + 3x_2 - x_3$$
s.t.

$$x_1 + x_3 \leq 2$$

$$\cancel{x_2 + x_1 \geq 1} \qquad -x_2 - x_1 \leq -1$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

set

$x_3^+ = x_3$, $x_3^- = 0$ $\qquad$ $x_3^+ = 0$, $x_3^- = -x_3$

$$x_3 = x_3^+ - x_3^- \qquad \Updownarrow$$

$$\max \quad x_1 + 3x_2 - x_3^+ + x_3^-$$
s.t.

$$x_1 + x_3^+ - x_3^- \leq 2$$
$$-x_2 - x_1 \leq -1$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$
$$x_3^+ \geq 0$$
$$x_3^- \geq 0$$