LINEAR PROGRAMMING

INTRODUCTION
Convex Optimization solves Gradient Descent

Linear Programming solves Convex Optimization

Simplex solves Linear Programming

Max Flow solves Simplex

Min Cut solves Max Flow

Matching solves Min Cut

2 Player Games solves Matching

0 Sum Games solves 2 Player Games

Quality explains Regularized Multiplicative Weights

Duality proves Follow the Leader

Solves Min Cut, Max Flow, Games
Linear Programming

Classic $4 a gallon profit

Premium $8 a gallon profit

Each week can brew

\[ \leq 2,000 \text{ gallons of classic} \]

\[ \leq 2,000 \text{ gallons of premium} \]

\[ \leq 3,000 \text{ gallons in total} \]

\[ X = \text{production per week of classic in 1,000s of gallons} \]

\[ Y = \text{production per week of premium in 1,000s of gallons} \]

maximize \[ 4X + 8Y \]

subject to

\[ X \leq 2 \]

\[ Y \leq 2 \]

\[ X + Y \leq 3 \]

\[ X \geq 0 \]

\[ Y \geq 0 \]
\[ x + y \leq 3 \]
\[ x \geq 0 \]
\[ y \geq 0 \]
\[ x \leq 2 \]
\[ y \leq 2 \]
maximize $x + 2y$

$7 = 2$

Feasible region of $x + y \leq 3$

$x \leq 2$

$y \leq 2$

$x \geq 0$

$y \geq 0$
$x + 7 = 3$

$7 = 2$

$x = 2$
Maximize \[ x + 2y - z \]

subject to
\[ x + y + z \leq 1 \]
\[ x \geq 0 \]
\[ y \geq 0 \]
\[ z > 0 \]

\[ x + 2y - z = 1 \]
\[ \cos \theta = 1 \]
\[ \frac{1}{2}, \frac{1}{4}, 0 \]
\[ x + y + z = 1 \]
\[ z = 0 \]
\[ y = 0 \]
\[ x = 0 \]
\[ x \geq 0 \]
\[ y \geq 0 \]
\[ z > 0 \]
\[ x + y + z \leq 1 \]
\[ \max \quad - \]
\[ \text{subject to} \]
\[ x_1 \geq 0 \]
\[ x_1 \leq 1 \]
\[ x_2 \geq 0 \]
\[ x_2 \leq 1 \]
\[ \vdots \]
\[ x_n \geq 0 \]
\[ x_n \leq 1 \]
cost function

\[
\text{maximize } \quad c_1 x_1 + c_2 x_2 + \cdots + c_n x_n
\]

constraints

\[
\begin{align*}
a_1^1 x_1 + a_2^1 x_2 + \cdots + a_n^1 x_n & \leq b_1 \\
a_1^2 x_1 + a_2^2 x_2 + \cdots + a_n^2 x_n & \leq b_2 \\
\vdots \\
a_1^n x_1 + a_2^n x_2 + \cdots + a_n^n x_n & \leq b_n
\end{align*}
\]

\[
x_1 \geq 0 \\
x_2 \geq 0 \\
\vdots \\
x_n \geq 0
\]
\[ \text{max} \quad x_1 + 3x_2 - x_3 \]
\[ \text{s.t.} \quad x_1 + x_3 \leq 2 \]
\[ -x_2 - x_1 \leq -1 \]
\[ x_1 \geq 0 \]
\[ x_2 \geq 0 \]
\[ x_3 = x_3^+ - x_3^- \]

\[ \text{max} \quad x_1 + 3x_2 - x_3^+ + x_3^- \]
\[ \text{s.t.} \quad x_1 + x_3^+ - x_3^- \leq 2 \]
\[ -x_2 - x_1 \leq -1 \]
\[ x_1 \geq 0 \]
\[ x_2 \geq 0 \]
\[ x_3^+ \geq 0 \]
\[ x_3^- \geq 0 \]